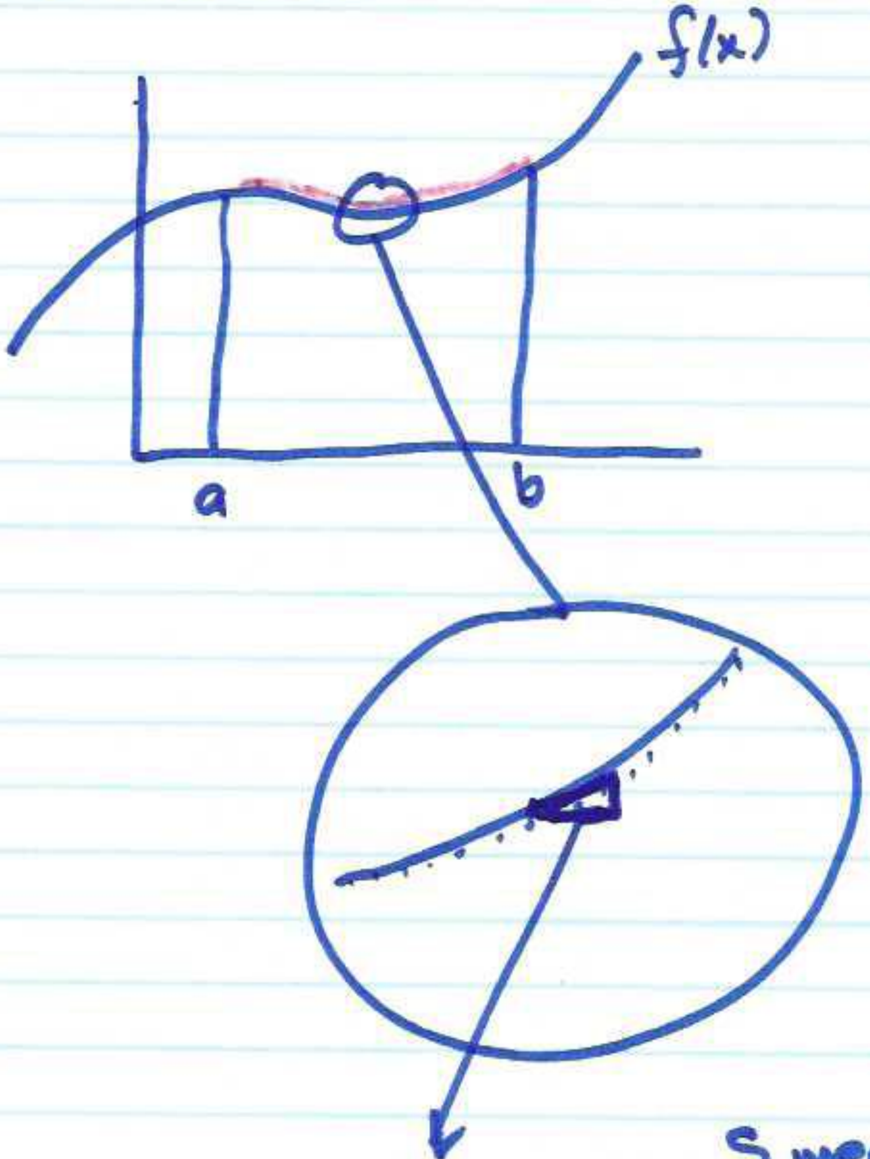
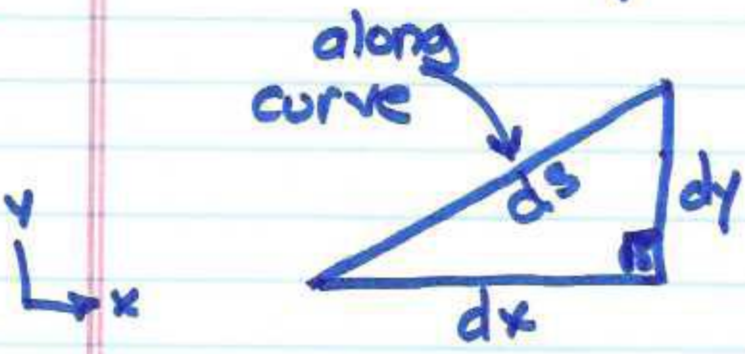
RA 7QC

early 8-27 1

Rectification



S means distance



$$\underline{ds^2 = dx^2 + dy^2}$$

2

$$ds = \sqrt{\frac{dx^2+dy^2}{dx^2}} \cdot dx = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_a^b \sqrt{1+(y')^2}\, dx$$

Ex: $y = f(x) = \frac{x^3}{12} + \frac{1}{x}$ $a=1$ to $b=4$

$y'(x) = \frac{x^2}{4} - \frac{1}{x^2}$ now

$$\cancel{\left(1+\frac{x^2}{4}-\frac{1}{x^2}\right)^2} \quad 1 + \left(\frac{x^2}{4} - \frac{1}{x^2}\right)^2$$

$$1 + \frac{x^4}{16} + 2\left(-\frac{1}{4}\right) + \frac{1}{x^4}$$

simplify $\frac{x^4}{16} + \frac{1}{2} + \frac{1}{x^4}$ = square?

3

$$\left(\frac{x^2}{4}+\frac{1}{x^2}\right)^2=\frac{x^4}{16}+\frac{1}{2}+\frac{1}{x^4}$$

In summary $1+(y')^2$ where $y'=\frac{x^2}{4}-\frac{1}{x^2}$

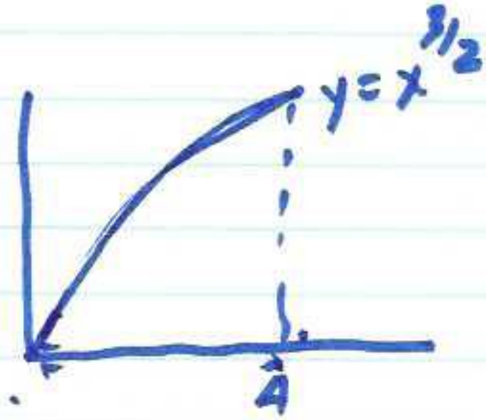
is $\left(\frac{x^2}{4}+\frac{1}{x^2}\right)^2$

$$\int_1^4\sqrt{\left(\frac{x^2}{4}+\frac{1}{x^2}\right)^2}\,dx =$$

$$\int_1^4\left(\frac{x^2}{4}+\frac{1}{x^2}\right)dx=\frac{1}{4}\int_1^4 x^2dx+\int_1^4\frac{dx}{x^2}$$

$$\left[\frac{x^3}{12}\right]_1^4+\left[-\frac{1}{x}\right]_1^4=\left(\frac{64}{12}-\frac{1}{12}\right)+\left(-\frac{1}{4}+1\right)$$

Ex: length of curve $y = x^{3/2}$ from $x=0$ to 4



$$y'(x) = \frac{3}{2}x^{\frac{1}{2}}$$

$$\int_0^4 \sqrt{1+\frac{9x}{4}}\,dx = \int_0^4 \sqrt{\frac{4+9x}{4}}\,dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{9x+4}\,dx$$

Let $u = 9x+4$ then $du = 9\,dx$

x-form: $\frac{1}{2}\int_{x=0}^{x=4} u^{1/2}\,\frac{du}{9} = \frac{1}{18}\int_{x=0}^{x=4} u^{1/2}\,du$

$$\Rightarrow \frac{1}{18}\cdot\frac{2}{3}\left[u^{3/2}\right] = \frac{1}{27}\left[(9x+4)^{3/2}\right]_0^4 = ?$$

$$\cancel{\frac{1}{27}(40-4) = \frac{36}{27} = \boxed{\frac{4}{3}}}$$

$$= \frac{1}{27}\left[40^{3/2} - 4^{3/2}\right]$$

~ 253 $\quad\quad 8$

approx.

$$\frac{1}{27}\cdot 245$$

⑥

$$y = \frac{x^3}{3} + x^2 + x + \frac{1}{4x+1} \qquad 0 \le x \le 2$$

$$y' = \frac{x^2}{} + 2x + 1 - \frac{4}{(4x+1)^2}$$

$$y = \frac{1}{2}(e^x + e^{-x}) \qquad -1 \le x \le 1$$

$y = \cosh x$

$y' = \sinh x$

$1 + (y')^2 = 1 + \sinh^2 x = \cosh^2 x$

$$\int_{-1}^{+1} \sqrt{\cosh^2 x}\, dx = \int_{-1}^{+1} \cosh x\, dx =$$

$$\Big[\sinh x\Big]_{-1}^{1} = \left[\frac{e^x - e^{-x}}{2}\right]_{-1}^{+1}$$

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$\frac{1}{2}\left[e^{1}-e^{-1}-\left[e^{-1}-e^{1}\right]\right)$

$\frac{1}{2}\left[2e-2e^{-1}\right] = e^{1}-e^{-1} = 2\sinh 1$

$x = \int_0^y \sqrt{\sec^4 t - 1}\, dt$ $\quad y(t)$

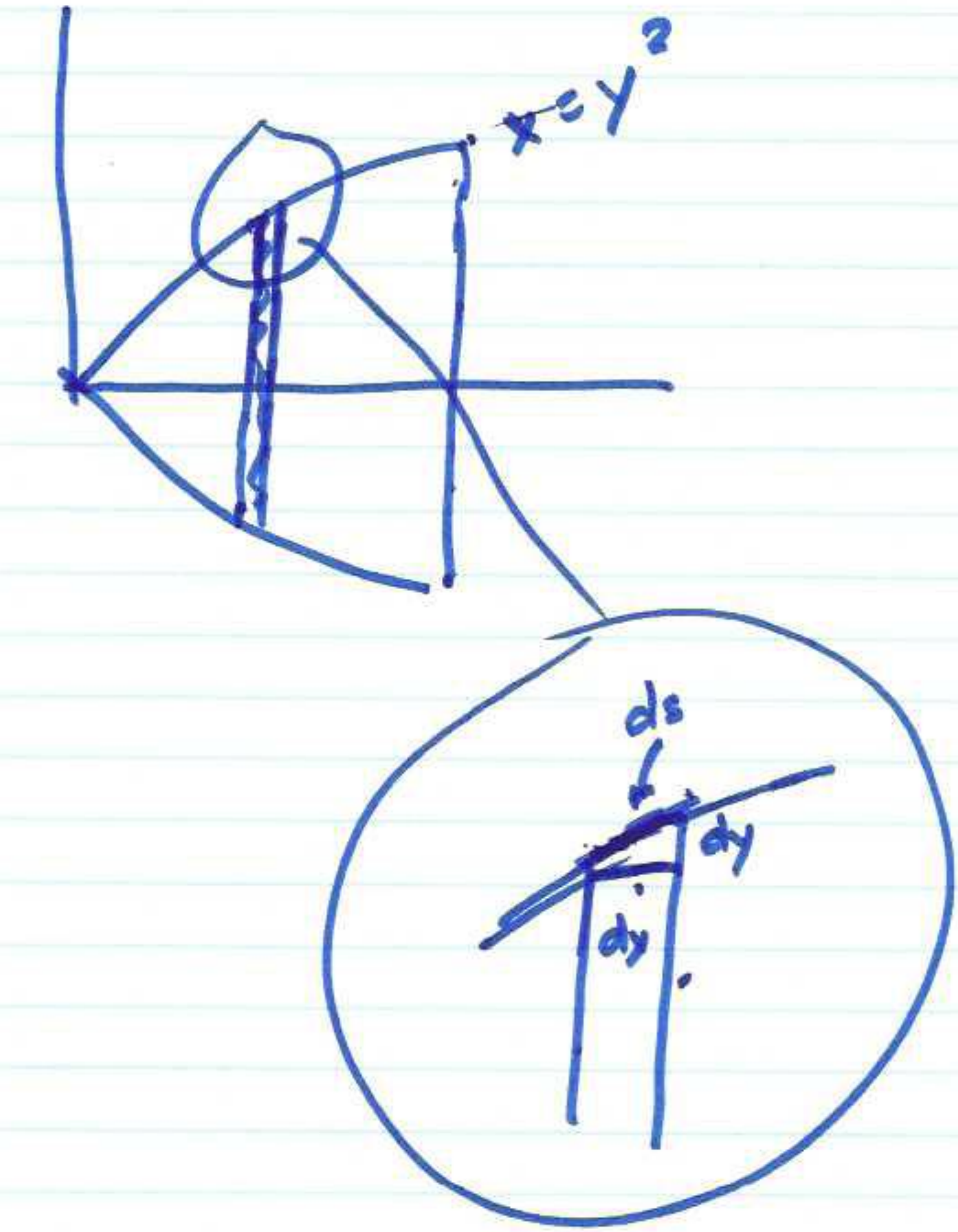
$-\frac{\pi}{4} \le y \le \frac{\pi}{4}$

$F'(x) = \left(\int_c^x f(t)\,dt\right)' = f(x)$

$x' = \sqrt{\sec^4 y - 1}$

$\sqrt{1+\left(\frac{dx}{dy}\right)^2} = \sqrt{1+\sec^4 y - 1} = \sec^2 y$

$\int_{-\pi/4}^{\pi/4} \sec^2 y\, dy = \left[\tan y\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = 2$

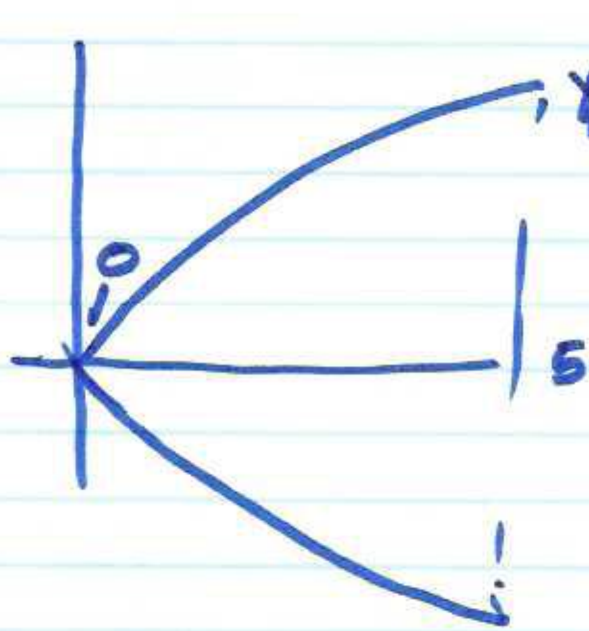


$x = y^2$
ds
dy
dy

$$\int_{\text{start}}^{\text{stop}} (ds)(2\pi)\overset{y}{f(x)} = \text{Surface Area}$$

$$ds = \sqrt{1+(y')^2}$$

$$S = 2\pi \int_a^b y \underbrace{\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx}_{ds}$$



(10)

$y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$$S = 2\pi \int_0^5 \sqrt{x}\sqrt{1 + \frac{1}{4x}}\, dx$$

$$= 2\pi \int_0^5 \sqrt{x + \frac{1}{4}}\, dx$$

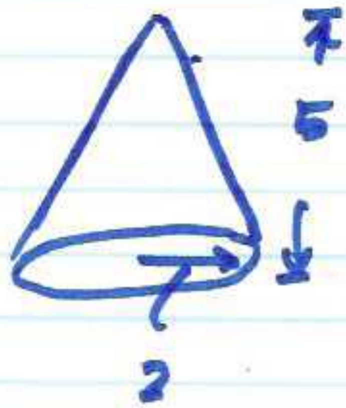
$u = x + \frac{1}{4} \quad du = dx$

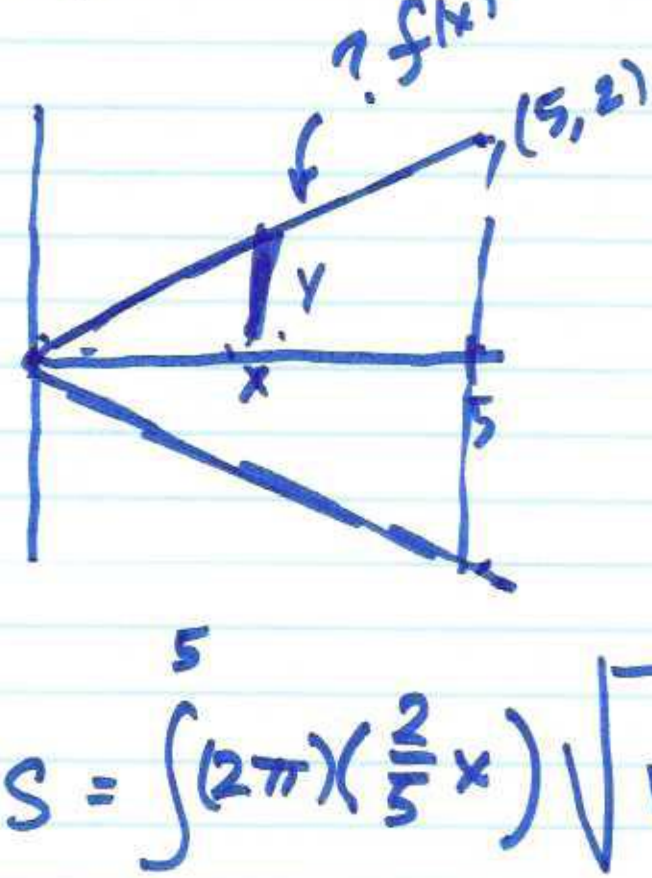
$$= 2\pi \int_{x=0}^{x=5} u^{1/2}\, du = 2\pi \left[u^{3/2} \right] \cdot \frac{2}{3}$$

$$\frac{4\pi}{3} \left[\left(x + \frac{1}{4}\right)^{3/2} \right]_0^5$$

$$\frac{4\pi}{3} \left[(5.25)^{3/2} - (0.25)^{3/2} \right]$$

Find surface area (excluding base) of a cone of height 5 and base radius 2





$$y = \frac{2}{5}x$$

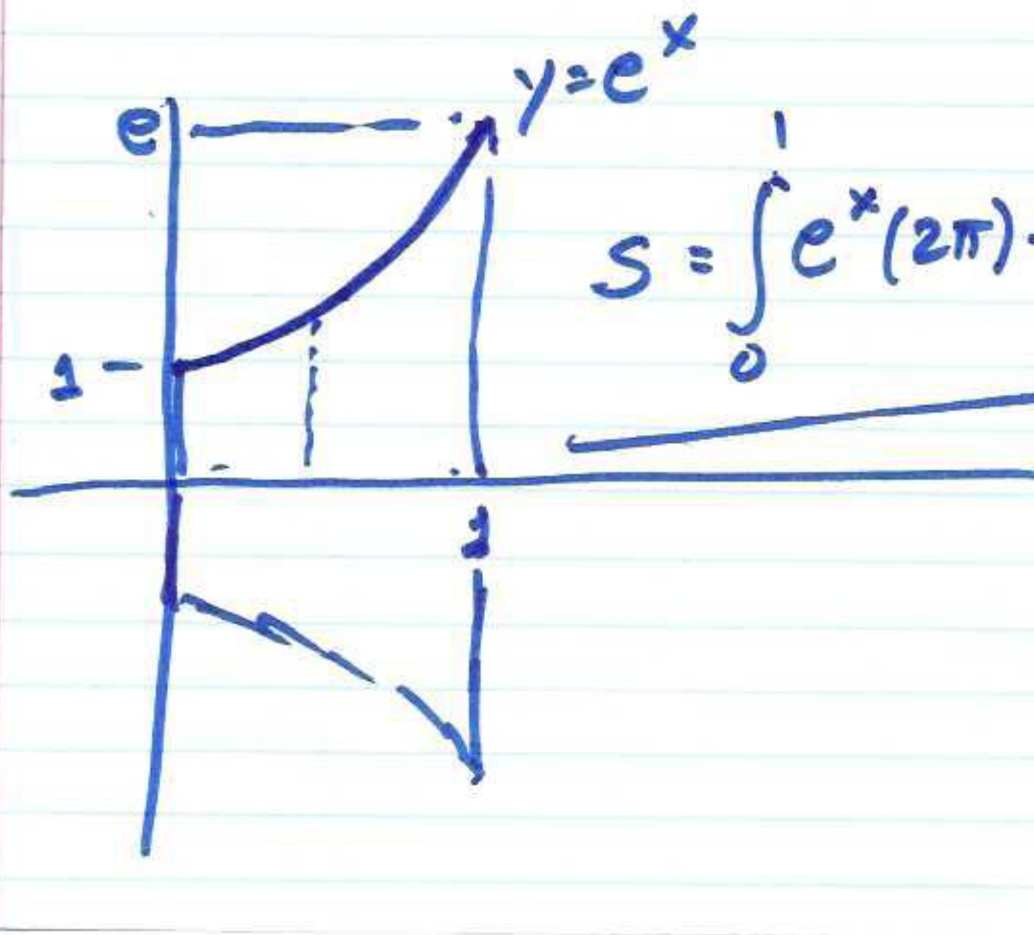
$$S = \int_0^5 (2\pi)\left(\frac{2}{5}x\right)\sqrt{1 + \frac{4}{25}}\ dx$$

(12)

$$\frac{4\pi}{5}\int_0^5 x\cdot\sqrt{\frac{29}{25}}\,dx$$

$$\frac{4\pi}{5}\cdot\frac{\sqrt{29}}{5}\int_0^5 x\,dx = \frac{4\pi\sqrt{29}}{25}\left[\frac{x^2}{2}\right]_0^5$$

$$= \frac{\cancel{4}^{2}\pi\sqrt{29}}{\cancel{25}}\cdot\frac{\cancel{25}}{\cancel{2}} = 2\pi\sqrt{29}$$



$$S = \int_0^1 e^x(2\pi)\cdot\sqrt{1+e^{2x}}\,dx$$

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$$S = 2\pi \int_{x=0}^{x=1} \cancel{u} \sqrt{1+u^2} \frac{du}{\cancel{u}}$$

$u = e^x$

$du = e^x dx$ so $dx = \dfrac{du}{e^x}$

$$S = 2\pi \int_{x=0}^{x=1} \sqrt{1+u^2}\, du$$

Let $u = \tan v$ NB: $1 + \tan^2 v = \sec^2 v$

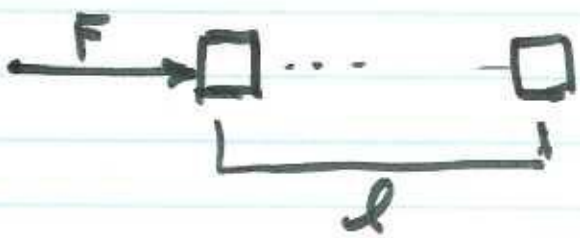
$du = \sec^2 v\, dv$

$$S = 2\pi \int_{x=0}^{1} \sqrt{\sec^2 v}\, \sec^2 v\, dv$$

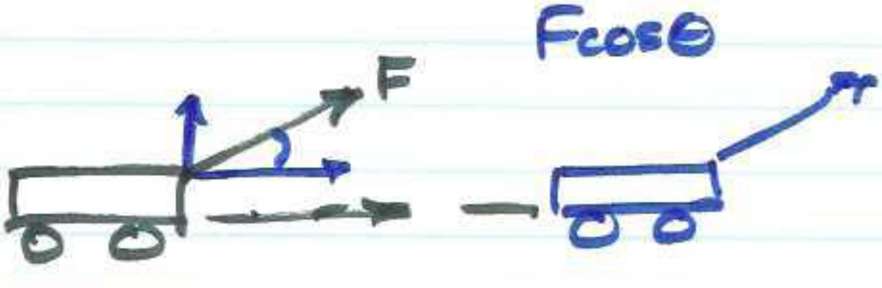
$$= 2\pi \int_{x=0}^{x=1} \sec^3 v\, dv \leftarrow \int$$

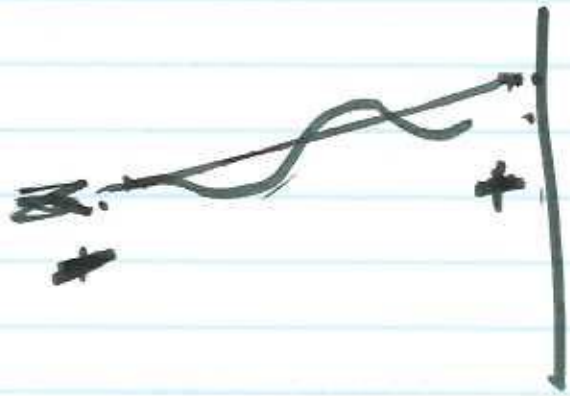
$$\int \sec^3 x\,dx = \frac{\sec x \tan x}{2} + \frac{1}{2}\int \sec x\,dx$$

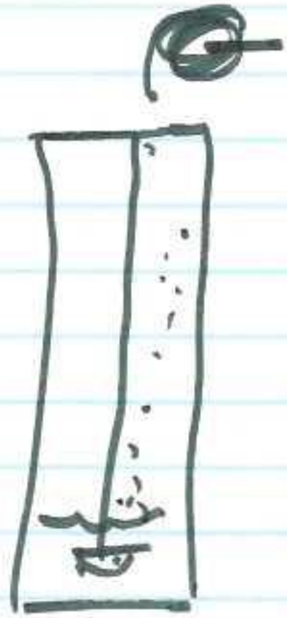
$$\int \sec x\,dx \quad \ln|\sec x + \tan x| + C$$



work done moving object is $F\ell$

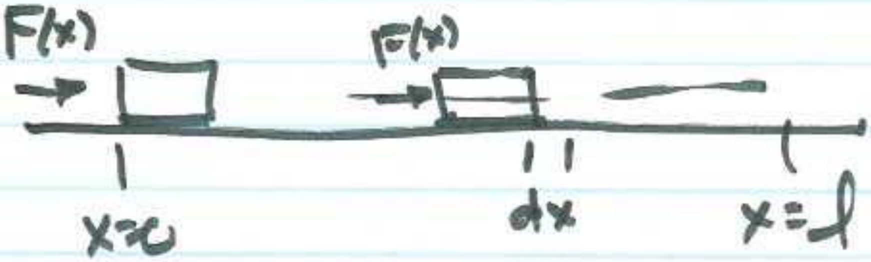






$$\int E \cdot dl$$

$$W = \int F(x)\,dx\,.$$



$$dW = F(x)\,dx$$