YL4QM 8-25 ①

$$\int \frac{\sin\sqrt{\theta}\, d\theta}{\sqrt{\theta \cos^3\sqrt{\theta}}}$$

Try $u = \cos\sqrt{\theta}$

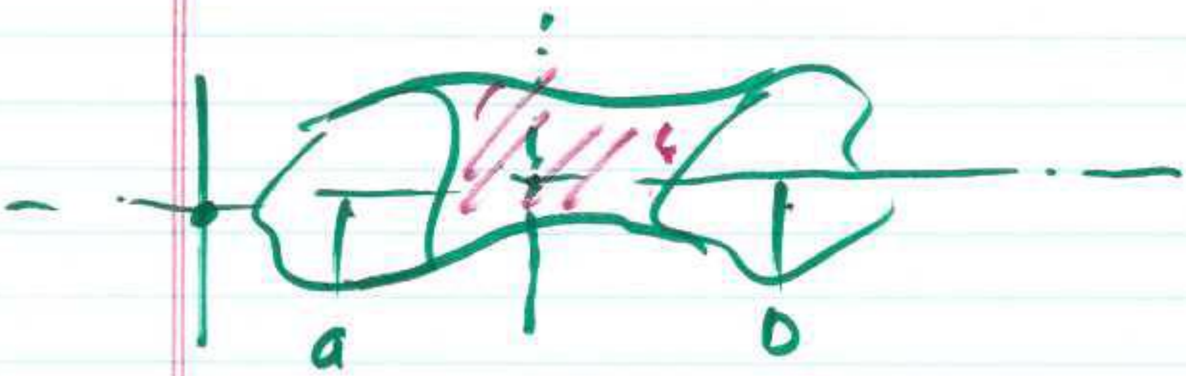
$$du = -\sin\sqrt{\theta} \cdot \frac{1}{2\sqrt{\theta}}\, d\theta$$

$$d\theta = \frac{-du}{\sin\sqrt{\theta}} \cdot 2\sqrt{\theta}\, du$$

$$\int \frac{\cancel{\sin\sqrt{\theta}}}{\cancel{\sqrt{\theta}}\sqrt{u^3}} \left[\frac{-du}{\cancel{\sin\sqrt{\theta}}} \cdot 2\cancel{\sqrt{\theta}}\right]$$

$$-2\int \frac{du}{\sqrt{u^3}} = -2\int u^{-3/2}\, du$$

$$+4\frac{1}{\sqrt{u}} + C$$



What is volume ?

$$dV = A(x)\,dx$$

$$Vol = \int_a^b dV = \int_a^b A(x)\,dx$$ Gen'l formula

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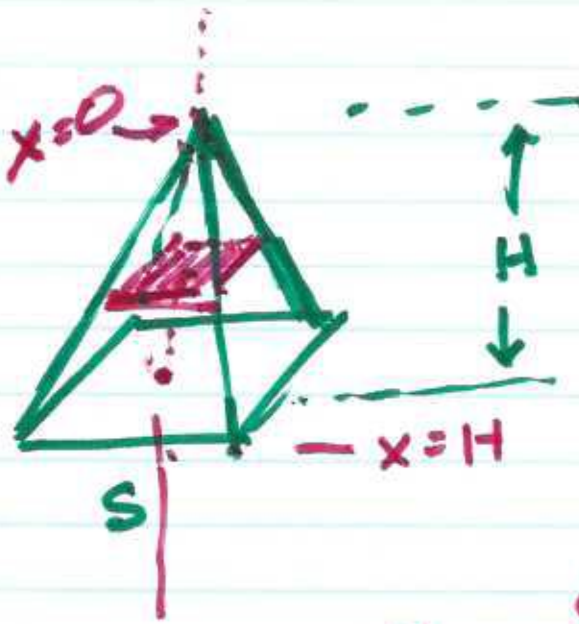


H = height

S = side of base

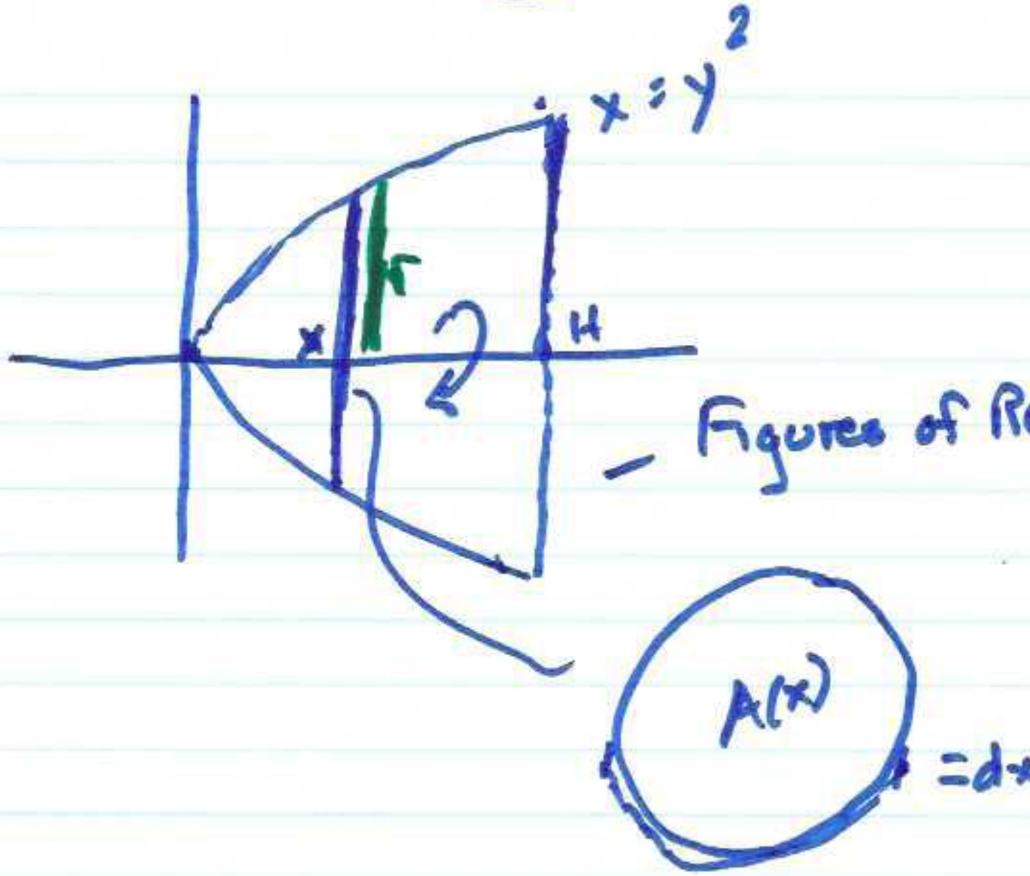
$$\frac{x}{H} = \frac{S(x)}{S}$$

$$\Rightarrow S(x) = \frac{xS}{H}$$

$$A(x) = [S(x)]^2 = \frac{x^2S^2}{H^2}$$

$$Vol = \int_0^H \frac{x^2S^2}{H^2}dx = \frac{S^2}{H^2}\left[\frac{x^3}{3}\right]_0^H$$

$$Vol = \frac{S^2}{H^2}\cdot\frac{H^3}{3} = \frac{1}{3}S^2H$$



– Figures of Revolution

radius of section @ x is $\sqrt{x}$

What is $A(x)$? $A(x) = \pi \left(\sqrt{x}\right)^2 = \pi x$

$dV(x) = \pi x \, dx \Rightarrow$

$$\text{Vol} = \int_0^H \pi x \, dx = \left[\frac{\pi x^2}{2}\right]_0^H = \frac{\pi H^2}{2}$$

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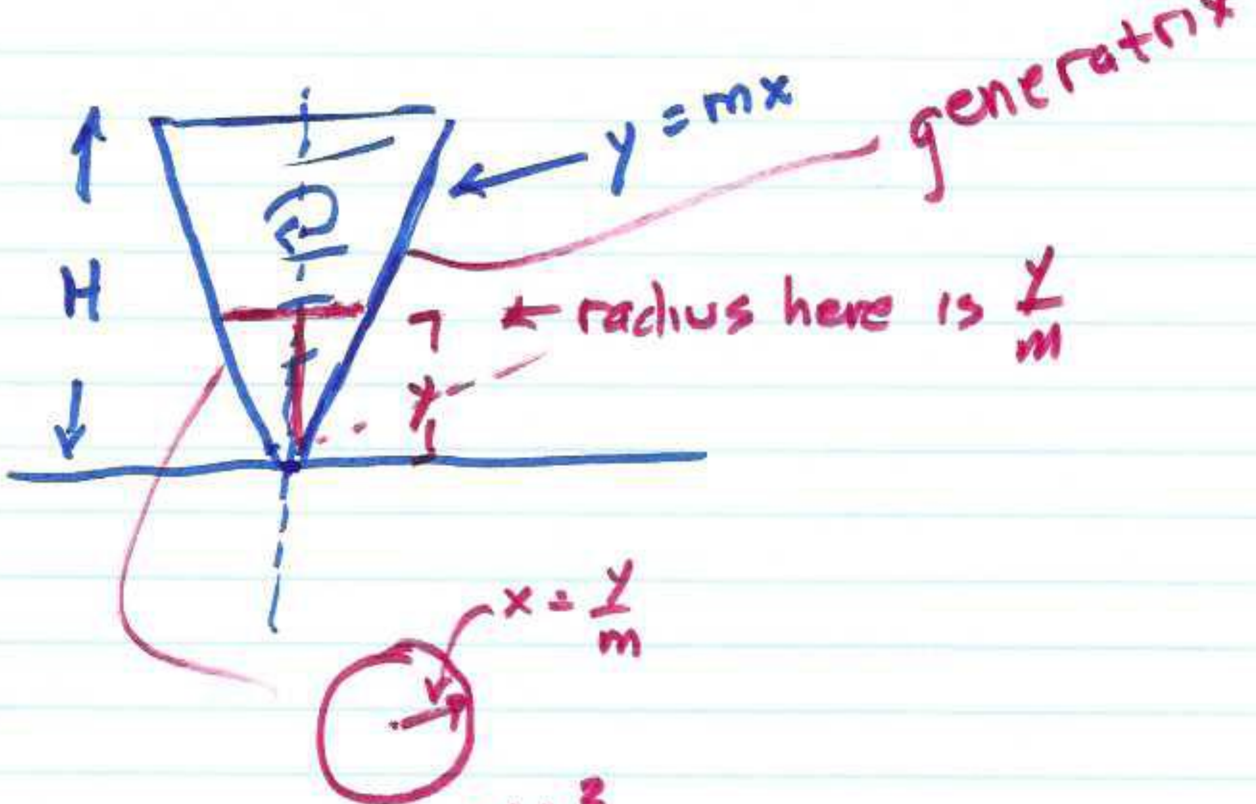
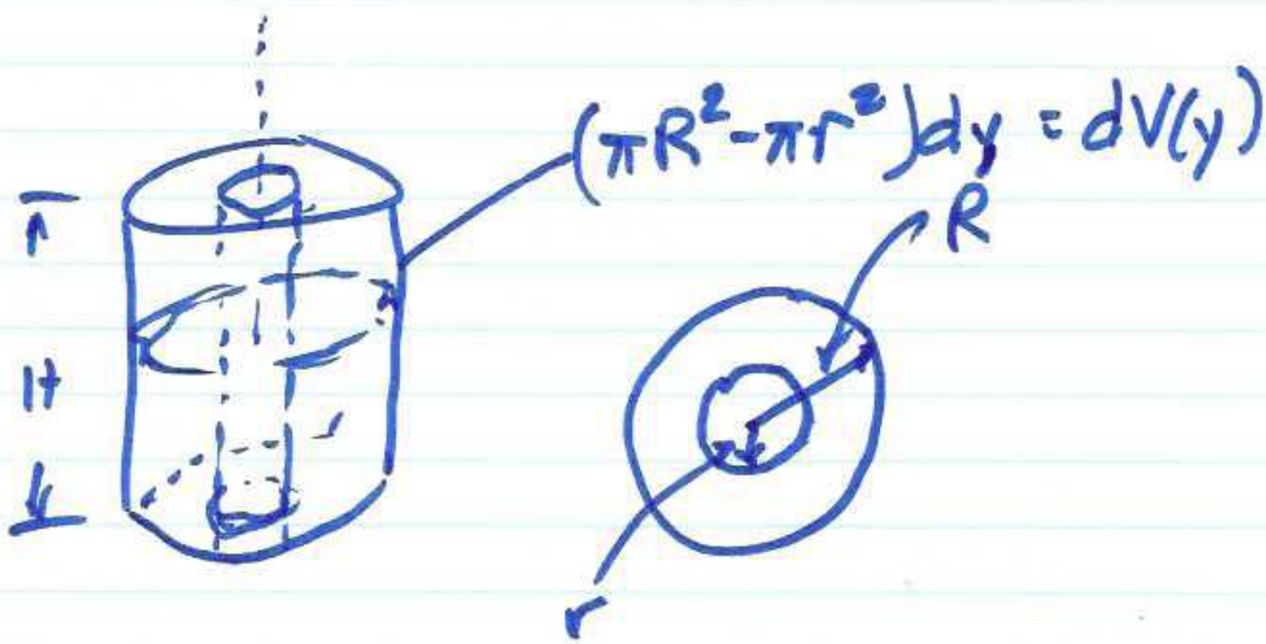


So $A(y) = \pi\left(\frac{y}{m}\right)^2$

$$\text{Vol} = \int_0^H A(y)\,dy = \int_0^H \frac{\pi y^2}{m^2}\,dy =$$

$$\frac{\pi}{m^2}\int_0^H y^2\,dy = \frac{\pi}{m^2}\left[\frac{y^3}{3}\right]_0^H$$

$$\text{Volume} = \boxed{\frac{\pi}{3m^2}\cdot H^3}$$



$\pi R^2 H$ = Vol outer

$\pi r^2 H$ = Vol inner

So figure has volume $\pi H(R^2 - r^2)$ ✓

$$V = \int_a^b A(x)\,dx = \int_a^b \underbrace{\pi\left[[R(x)]^2 - [r(x)]^2\right]}_{\substack{dA(x)\\ \text{of washer}}} dx$$

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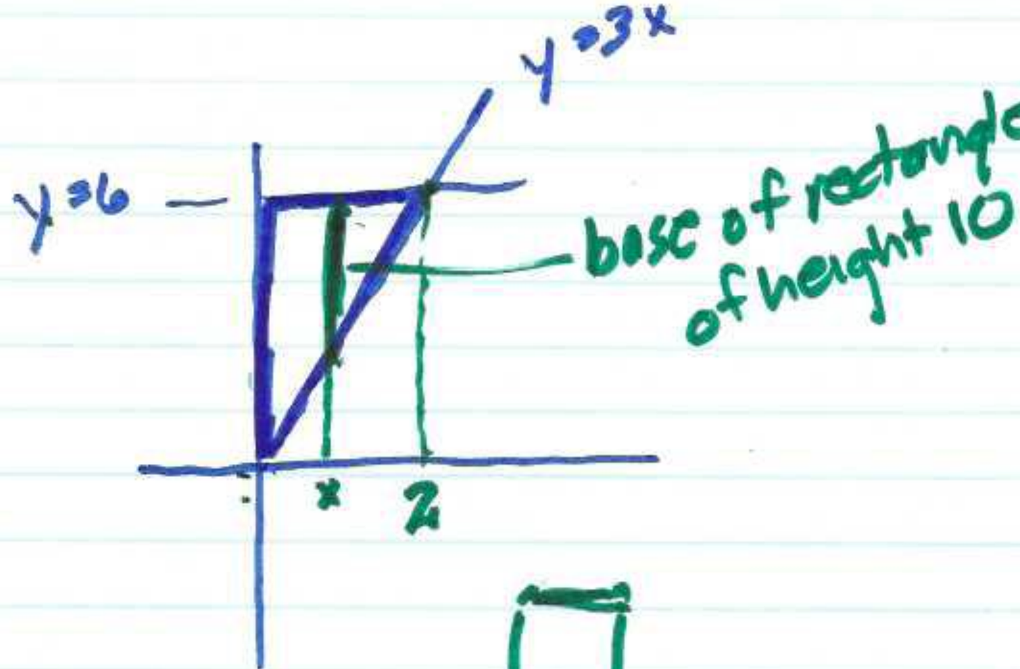

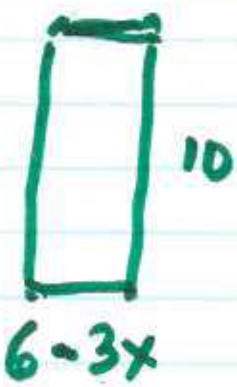

$A(x) = (6-3x)(10) = 60 - 30x$

So $dV(x) = A(x)\,dx = (60-30x)\,dx$

$$\text{Vol} = \int_0^2 (60-30x)\,dx$$

$$= \left[60x - 30\frac{x^2}{2}\right]_0^2 = 120 - 60 = 60$$

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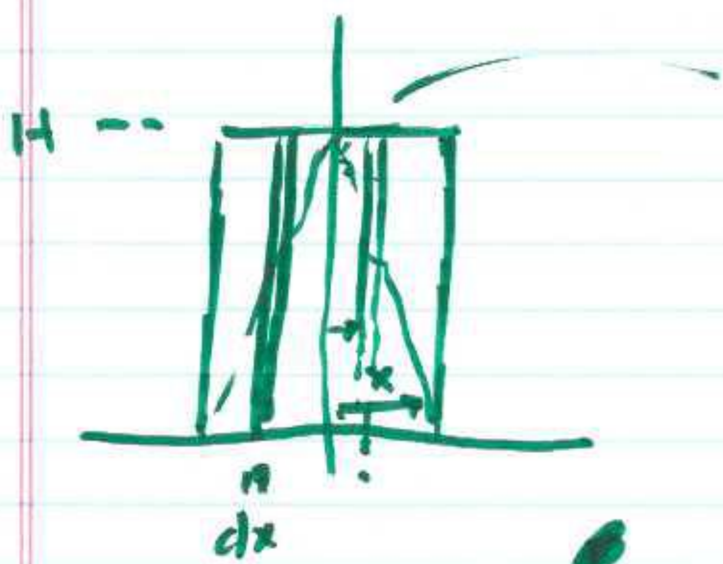
Cylinder height = H radius = R

So volume = $\pi R^2 H$



$(H)(2\pi x\,dx)$

$2\pi x H\,dx$ is

diff volume of cyl shell (tube) inside

$$Vol = \int_0^R 2\pi x H\,dx$$

$$= 2\pi H\left[\frac{x^2}{2}\right]_0^R = \cancel{2}\pi H\frac{R^2}{\cancel{2}}$$

$$= \pi R^2 H$$

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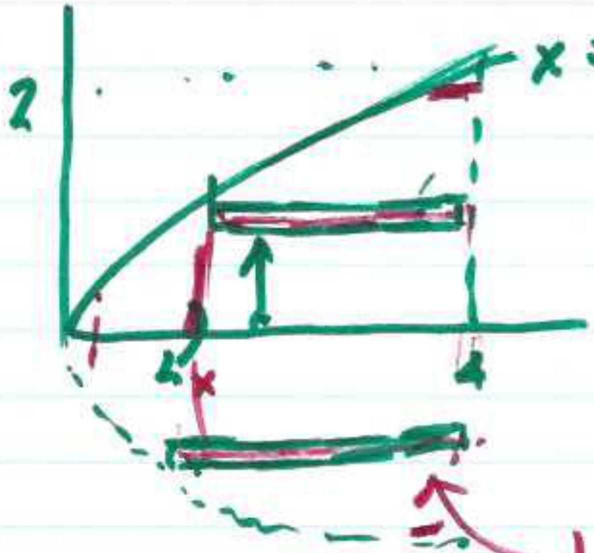


$y = \sqrt{x}$

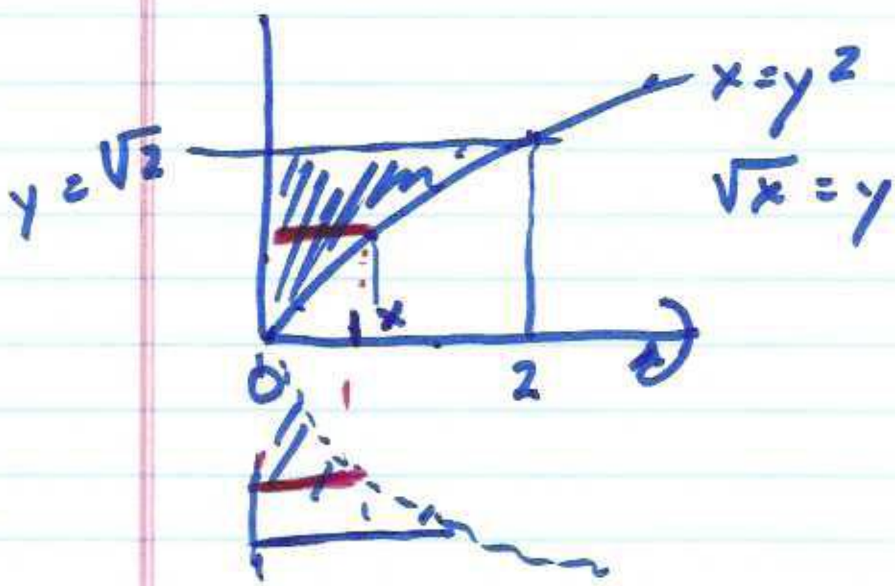
height $= 4 - y^2 = 4 - x$

radius $= \sqrt{x}$

$$\underbrace{\underbrace{2\pi\sqrt{x}\,dx}_{dA(x)} \cdot (4-x)}_{dV(x)}$$

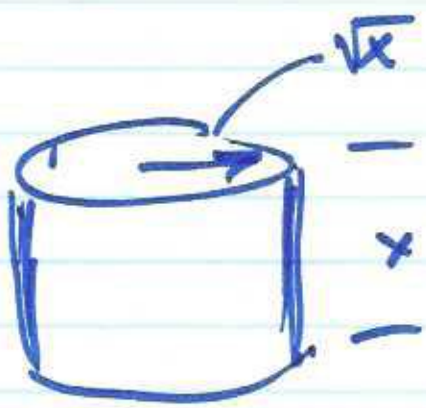
$$\text{Vol} = \int_0^4 2\pi\sqrt{x}\,(4-x)\,dx$$

$$= 2\pi\int_0^4 4\sqrt{x}\,dx - 2\pi\int_0^4 x^{3/2}\,dx$$



radius of shell @ x is $\sqrt{x}$

height of shel@ x is x



perimeter of shell is $2\pi\sqrt{x}$

so... $dV(x)$ $2\pi\sqrt{x}\,dx \cdot x$

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$$Vol = \int_0^2 2\pi x^{3/2}\,dx$$

$$= 2\pi\left(\frac{2}{5}x^{5/2}\right)\Big|_0^2$$

$$= \frac{4\pi}{5}\,2\sqrt{8} = \frac{4\pi}{5}\cdot 4\sqrt{2}$$

$$\frac{16\sqrt{2}\pi}{5}$$

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