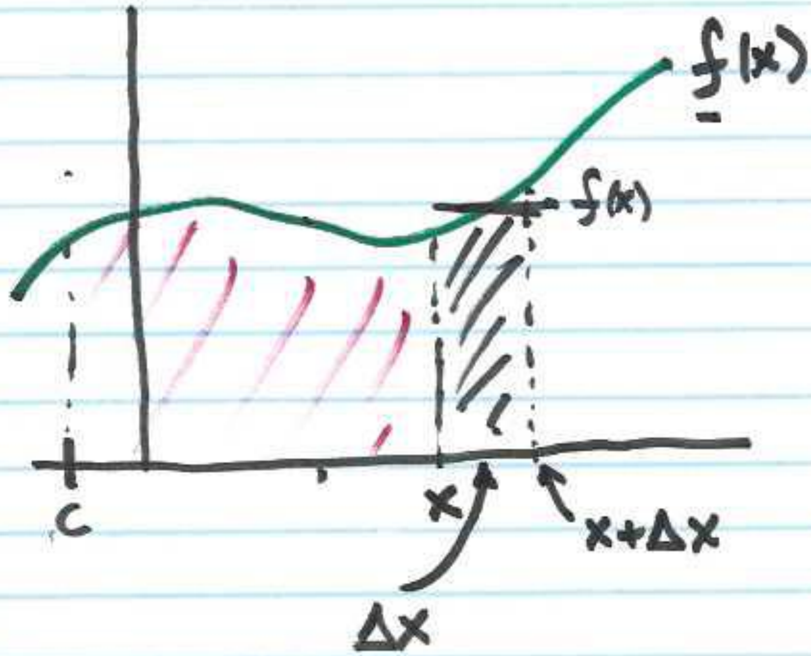
UNLWN $\underline{3:30}$ 8.-20

(1)

$\underline{F(x)}$ = area to right of some fixed point c



rate of change of area $\longrightarrow$ $\dfrac{F(x+\Delta x) - F(x)}{\Delta x}$

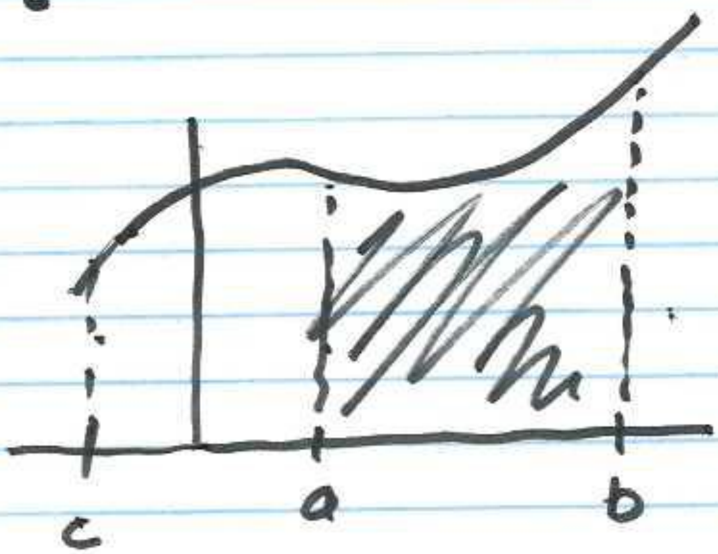
?

$f(x) \cdot \Delta x$

$$\frac{f(x)\cancel{\Delta x}}{\cancel{\Delta x}} = f(x)$$

$F'(x) = f(x)$ so...

$$F(x) = \int_c^x f(x)\,dx$$



SO.. $F(b) - F(a)$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Method of Substitution

$\int f(x)\,dx$ — can't see how to do

$x = g(y)$ $\quad$ $\underline{dx = ?\ g'(y)\,dy}$

$$\frac{dx}{dy} = g'(y)$$

$$\int \overbrace{f(g(y))}^{f(x)} g'(y)\,dy$$

$$\int f(x)\,dx = \int f(g(y))\,g'(y)\,dy$$

in terms of x $\qquad$ in term of y

where $x = g(y)$

Ex: $\int x e^{x^2} dx$

Sol: Let $y = x^3$ $\quad \frac{dy}{dx} = 2x \Rightarrow dy = 2x\,dx$

$$x\,dx = \frac{dy}{2}$$

$$\int e^y \left(\frac{dy}{2}\right) = \frac{1}{2}\int e^y dy = \boxed{\frac{1}{2}e^y + C}$$

$$= \frac{1}{2}e^{x^2} + C$$

---

① $\int 2(2x+A)^5 dx$

Try: $y = 2x + A \quad dy = 2dx$

$$\int 2(y^5)\frac{dy}{2} = \int y^5 dy = \frac{y^6}{6} + C$$

(2) $\int \sec(2t)\tan(2t)\,dt = ?$

$y = 2t \Rightarrow dy = 2dt \quad dt = \frac{dy}{2}$

$\int \sec y \tan y \,(\;)\frac{dy}{2}$

$\frac{1}{2}\int \sec y \tan y \,dy = ?$

$\frac{1}{2}\sec y + C \rightsquigarrow \boxed{\frac{1}{2}\sec(2t) + C}$

(3) $\int \sqrt{x}\,\sin^2(x^{3/2} - 1)\,dx$

$y = x^{3/2} - 1 \quad \frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$dy = \frac{3}{2}\sqrt{x}\,dx$

(6)

$$dx = \frac{2}{3}\frac{dy}{\sqrt{x}}$$

$$\int \cancel{\sqrt{x}} \sin^2(y)\left[\frac{2}{3}\frac{dy}{\cancel{\sqrt{x}}}\right]$$

$$\frac{2}{3}\int \sin^2 y \, dy$$

Note: $\sin \frac{z}{2} = \sqrt{\frac{1-\cos z}{2}}$

$$\Rightarrow \sin^2 y = \frac{1-\cos 2y}{2}$$

$$\Rightarrow \frac{\cancel{2}}{3}\int\left(\frac{1}{\cancel{2}}dy\right) - \frac{\cancel{2}}{3}\int \frac{1}{\cancel{2}} \cos 2y \, dy$$

$$\downarrow$$

$$\frac{y}{3} - \frac{1}{3}\frac{\sin(2y)}{2} = \frac{y}{3} - \frac{1}{6}\sin(2y) + C$$

4) $\int \cos x \, e^{\sin x} dx$

Let $y = \sin x$ then $dy = \cos x \, dx$

so $dx = \frac{dy}{\cos x}$ and...

$$\int \cancel{\cos x} \, e^y \frac{dy}{\cancel{\cos x}} = \int e^y dy = e^y + C$$

$$\boxed{e^{\sin x} + C}$$

Z-score

5) $\int \frac{dx}{\ln x}$, $\int e^{-x^2} dx$ fail

6) $\int \frac{dx}{x \ln x}$ Set $y = \ln x$ $dy = \frac{dx}{x}$

$$\int \frac{dy}{y} = \ln y + C \rightarrow \boxed{\ln(\ln x) + C}$$

$\underline{\ln t^{1/2} = \frac{1}{2}\ln t}$

7) $\int \frac{\ln\sqrt{t}}{t}\,dt$

$\int \frac{\frac{1}{2}\ln t}{t}\,dt \leadsto \frac{1}{2}\int \frac{\ln t}{t}\,dt$

$d(\ln t) = \frac{dt}{t}$ so let $y = \ln t$

$dy = \frac{dt}{t}$

$\int \frac{\ln\sqrt{t}}{t}\,dt = \frac{1}{2}\int y\,dy = 2$

$\frac{1}{2}\frac{y^2}{2} + C \rightarrow \underline{\frac{1}{4}(\ln t)^2 + C}$

9

$$\int \frac{\sin\sqrt{\theta}\ d\theta}{\sqrt{\cos^3\sqrt{\theta}}}$$

$$\frac{d\sqrt{\theta}}{d\theta} = \theta^{-1/2}$$

$$\frac{1}{\sqrt{\theta}}$$

Try: $y = \cos\sqrt{\theta}$

$$dy = -\sin\sqrt{\theta} \cdot \frac{1}{\sqrt{\theta}}\, d\theta$$

$$\cancel{\int \sin\sqrt{\theta}}$$

$$\boxed{d\theta = \sqrt{\theta}\,(-\sin\sqrt{\theta})\, dy}$$

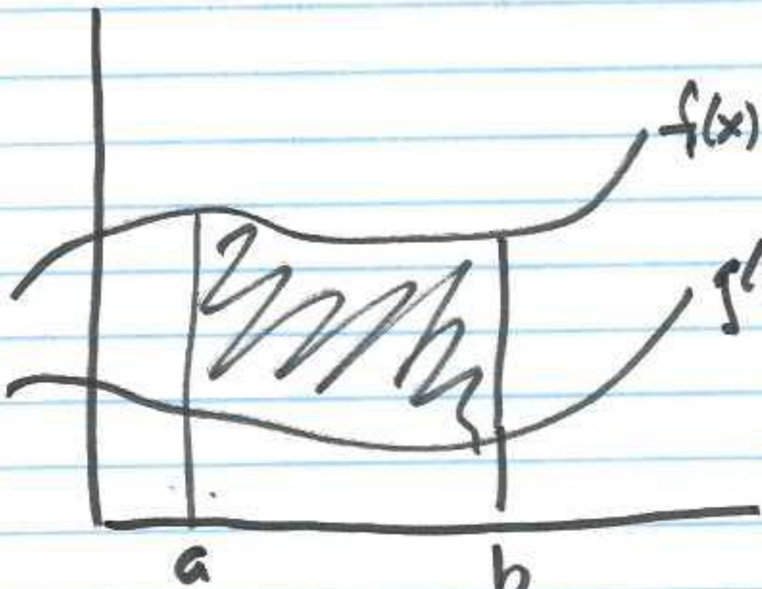
$$\cancel{\int \frac{\quad}{\sqrt{y^{3/2}}}}$$

$$d\theta = dy\left[\sqrt{\theta} \cdot \frac{1}{-\sin\sqrt{\theta}}\right]$$

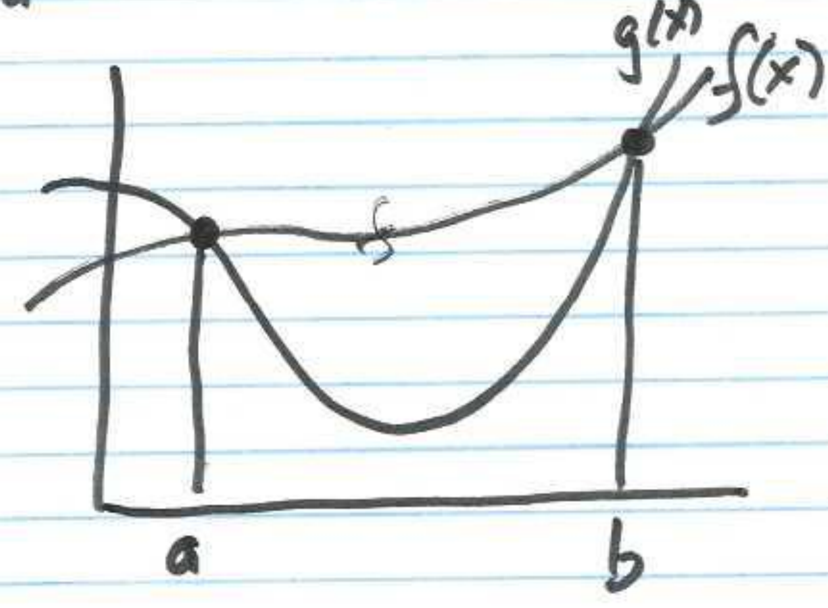
$$\int \frac{\sin\sqrt{\theta}}{-\sin\sqrt{\theta}} \left(\sqrt{\theta}\right) \cdot \frac{1}{\sqrt{y^{3/2}}}\, dy$$

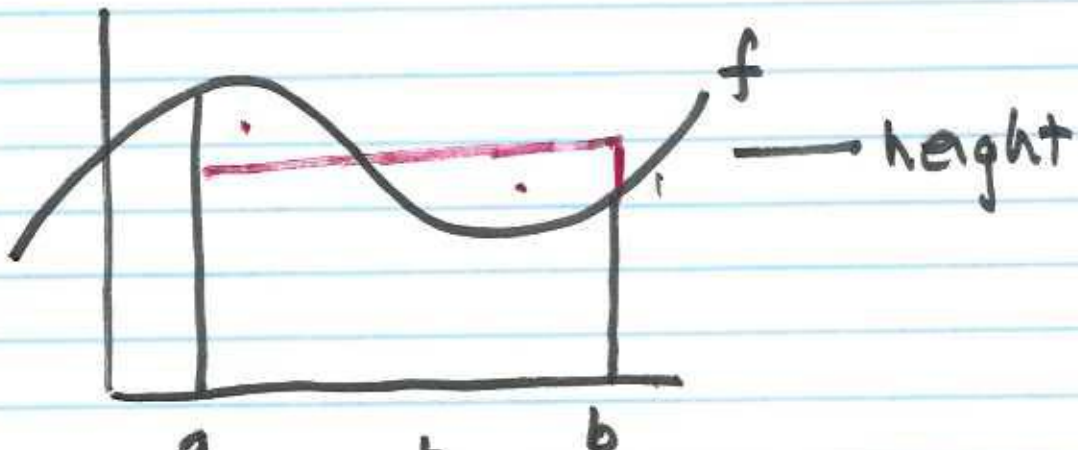
-1

!!



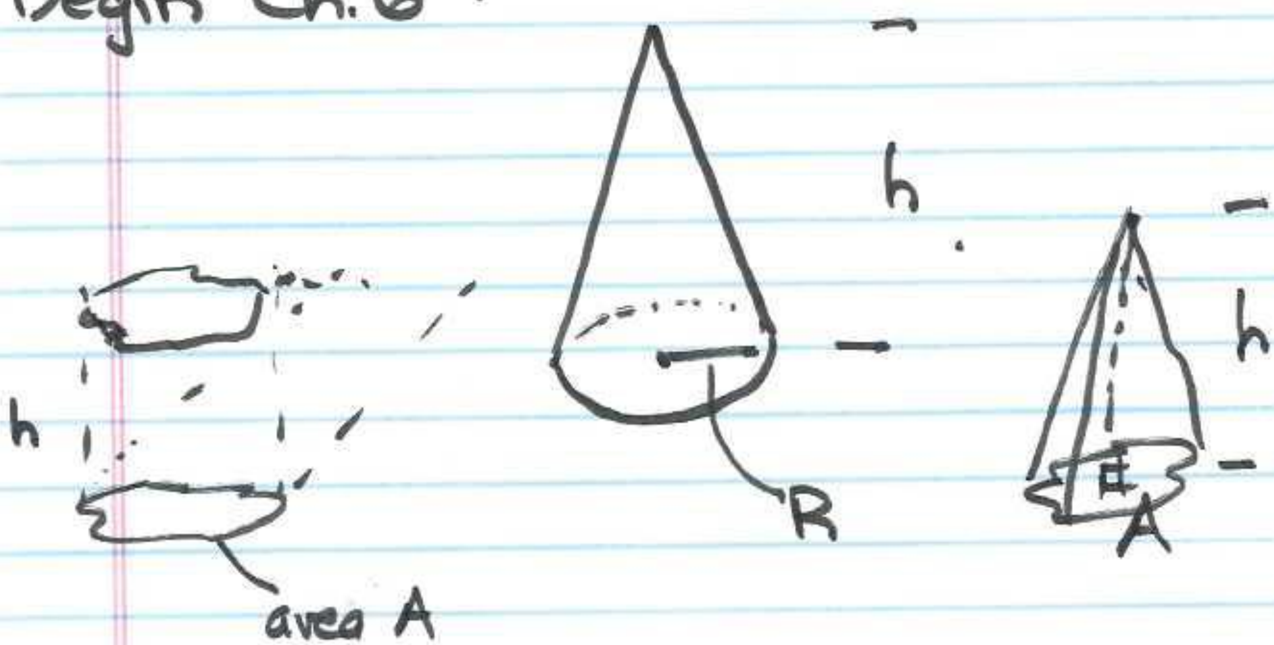
$$\int_a^b \left[f(x) - g(x)\right] dx = \text{area between curves}$$

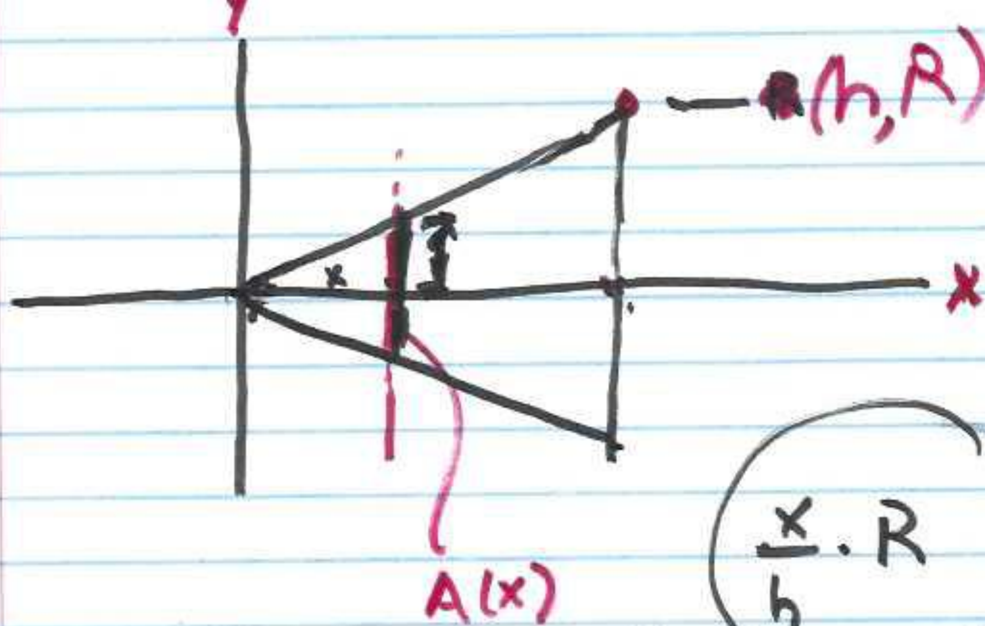




$$\hat{f}_{[a,b]} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

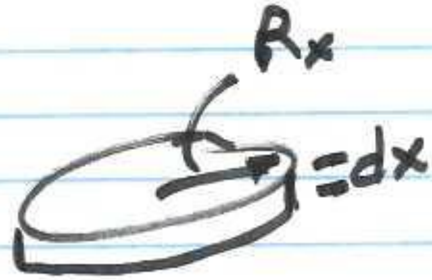
Begin Ch. 6 :

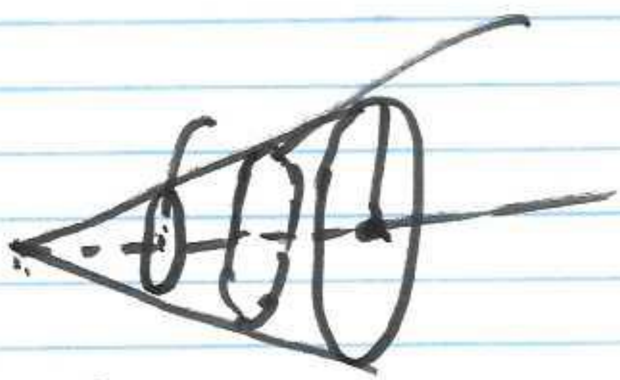




$\frac{x}{h}\cdot R$

$$A(x) = \pi\left(\frac{xR}{h}\right)^2$$





$$dV(x) = A(x)\cdot dx$$

$$Vol = \int_0^h \frac{\pi x^2 R^2}{h^2}\,dx = \frac{\pi R^2}{h^2}\int_0^h x^2\,dx$$

$$\frac{\pi R^2}{h^2}\int_0^h x^2\,dx = \frac{\pi R^2}{h^2}\cdot\left[\frac{x^3}{3}\right]_0^h = \frac{\pi R^2}{h^2}\cdot\frac{h^3}{3} =$$

$$\text{Vol} = \frac{\pi R^2}{3}\cdot h$$

This is the "Method of Disks"