KJCLT

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①

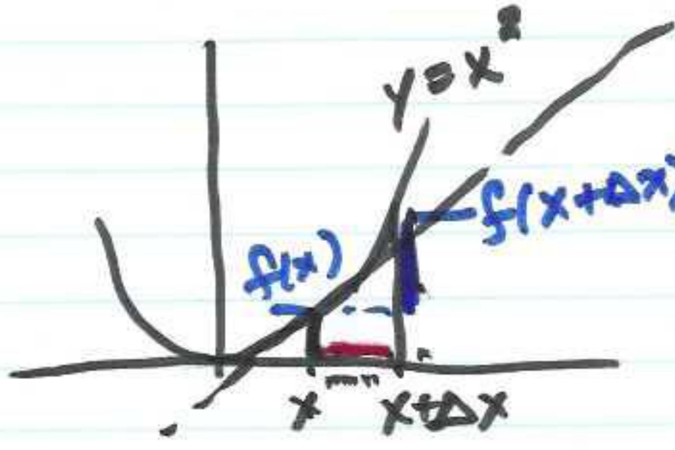
Derivative:

$f(x)$, find derivative

$f'(x)$, $\frac{df}{dx}$, $D_x f$

$$f'(x) = \lim_{\Delta x \to 0} \left[\frac{f(x+\Delta x) - f(x)}{\Delta x} \right]$$

Δx is an "increment" of x



$f(x) = x^2$ $\quad f(x+\Delta x) = x^2 + 2x\Delta x + (\Delta x)^2$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{x^2 + 2x\cancel{\Delta x} + (\cancel{\Delta x})^2 - x^2}{\cancel{\Delta x}}$$

$$= 2x + \Delta x$$

What is limit of $2x+\underbrace{\Delta x}_{0}$ as $\Delta x \to 0$

$f(x)=x^2$ | $f'(x)=2x$

$f(x)$	$f'(x)$
c, constant	0
x	1
x^2	$2x$
x^3	$3x^2$
x^n	nx^{n-1} $\quad x^{n-1}$
x^π	$\pi x^{\pi-1}$
r rational $\quad x^r$	rx^{r-1}
$\sqrt{x}$	$\frac{1}{2}\cdot x^{-\frac{1}{2}} \to \frac{1}{2\sqrt{x}}$ $\quad x^{1/2}$

	$f(x)$	$f'(x)$
	e^x	e^x
	2^x	$(\ln 2)2^x$
$2 = e^{\ln 2}$	$(e^{\ln 2})^x$	
	$= e^{x \ln 2}$	$e^{x\ln 2}\cdot(\ln 2)$
	x^x	
$\cancel{x(x^{x-1})}$	$x = e^{\ln x}$	
	$x^x = e^{x \ln x}$	$x^x\left[\ln x + \overbrace{x\cdot\frac{1}{x}}^{1}\right]$
	$\ln x$	$1/x$
$g(x)$ diff	$e^{g(x)}$ ★	$e^{g(x)}\cdot g'(x)$
e^{x^2}	$\ln g(x)$	$\frac{1}{g(x)}\cdot g'(x) = \frac{g'(x)}{g(x)}$

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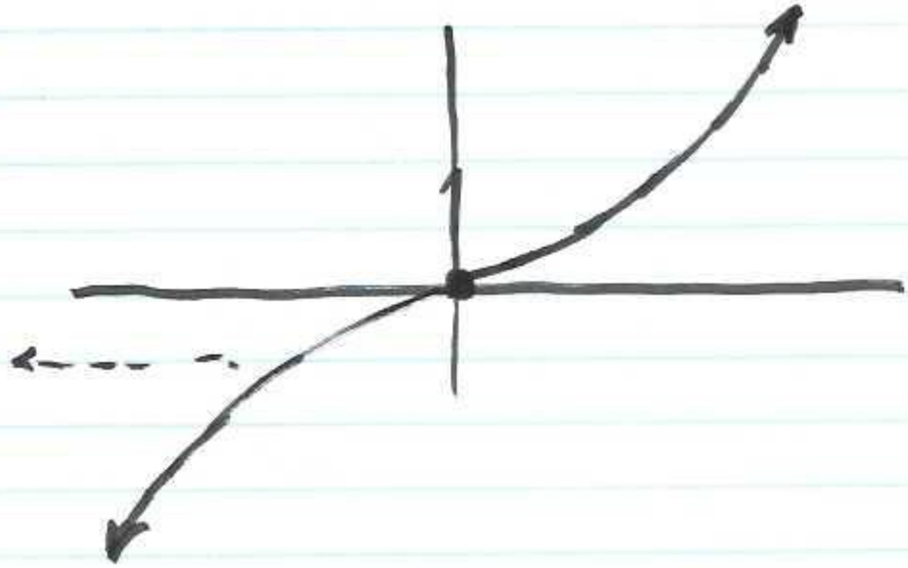
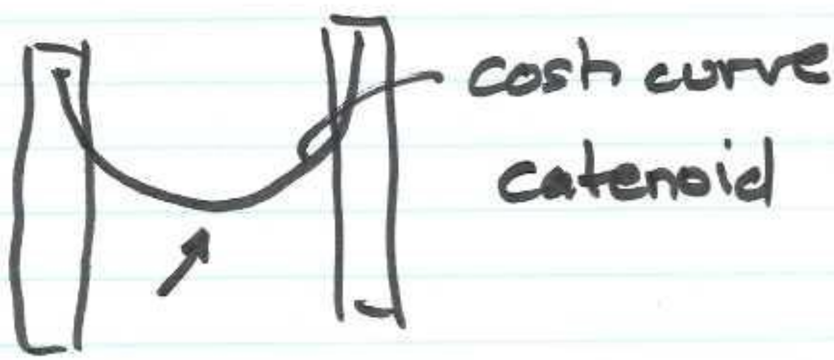
	$f(x)$	$f'(x)$
	$\sin x$	$\cos x$
	$\cos x$	$-\sin x$
	$\tan x$	$\sec^2 x$
$\text{ctn}\,x$:	$\cot x$	$-\csc^2 x$
	$\sec x$	$\sec x \tan x$
	$\csc x$	$-\csc x \cot x$
$\frac{e^x-e^{-x}}{2} =$	$\sinh x$	
$\frac{e^x+e^{-x}}{2} =$	$\cosh x$	$\frac{e^x-e^{-x}}{2} = \sinh x$
	$\tanh x$	
	$\coth x$	
	$\text{sech}\, x$	
	$\text{csch}\, x$	

$$y = \frac{e^x - e^{-x}}{2}$$





Rules:

Product Rule: $f(x) = g(x) \cdot h(x)$

$$f'(x) = g'(x)h(x) + g(x)h'(x)$$

Quotient Rule: $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$

$$f(x) = g(x) \cdot \frac{1}{h(x)} \qquad \frac{1}{h(x)} = [h(x)]^{-1}$$

$$f'(x) = g'(x) \cdot \frac{1}{h(x)} + g(x) \cdot (-1)[h(x)]^{-2} \cdot h'(x)$$

$$= \frac{g'(x)}{h(x)} - \frac{g(x)}{[h(x)]^2} \cdot h'(x)$$

$$\frac{h(x)g'(x)}{[h(x)]^2} - \frac{g(x)h'(x)}{[h(x)]^2}$$

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Chain Rule:

Composition of f & g is written $\underset{\text{post}}{f}\circ\underset{\text{pre}}{g}$

$$\left[(f\circ g)(x)\right]' = \left[f(g(x))\right]' = f'(g(x))\cdot g'(x)$$

$$\left[(\sin x)^{\cos x}\right]'$$

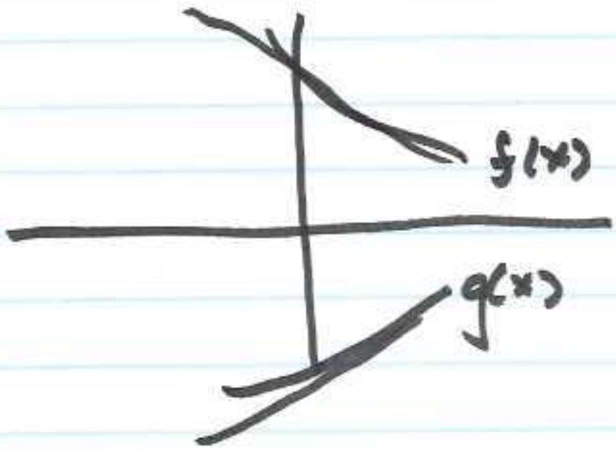
$$\sin x = e^{\ln \sin x}$$

$$\left(e^{\ln \sin x}\right)^{\cos x} = e^{\overbrace{\cos x \ln \sin x}}$$

$$(\sin x)^{\cos x}\cdot\left[-\sin x\cdot \ln \sin x + \cos x\cdot\frac{\cos x}{\sin x}\right]$$

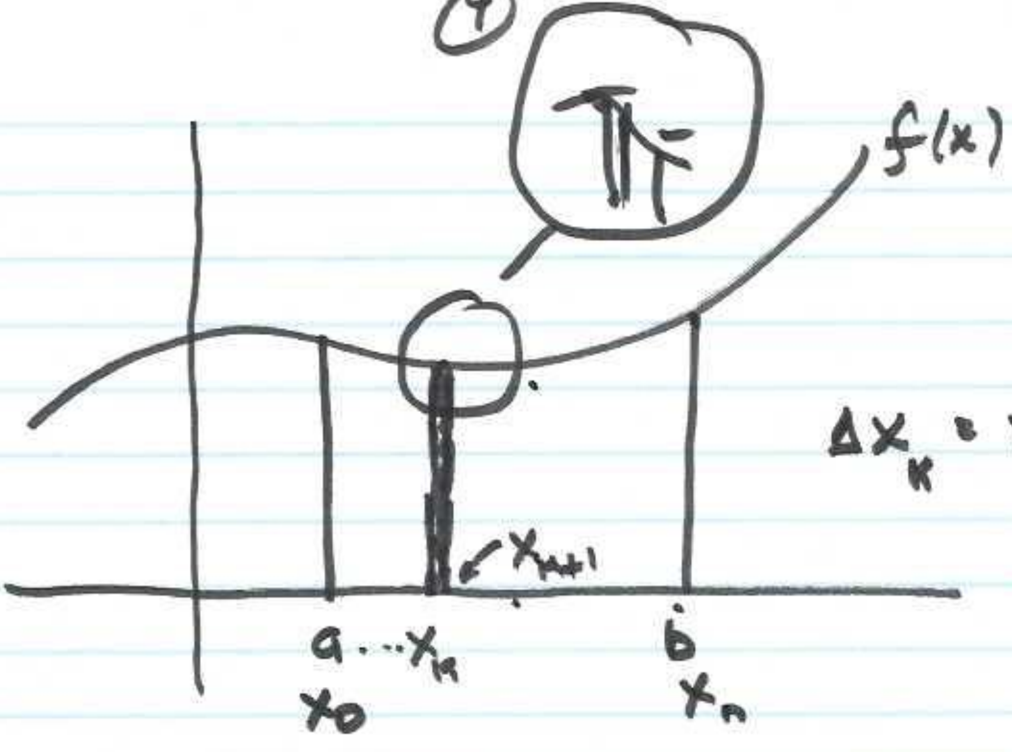
$$\left[(\sin x)^{\cos x}\right]' = (\sin x)^{\cos x}\left[\cos x\cot x - \sin x \ln \sin x\right]$$

If $f(x) > g(x)$ is $f'(x) > g'(x)$ NO



Linearity of Differentiation

$$\ast \quad \frac{d}{dx}\left(a f(x) + b g(x)\right) = a \frac{df}{dx} + b \frac{dg}{dx}$$



$$\Delta x_k = x_{k+1} - x_k$$

$$\text{Area}_k \approx \Delta x_k \cdot f(x_k)$$

$$\text{Area}_{\text{TOTAL}} = \sum_{k=0}^{n-1} \Delta x_k \, f(x_k)$$

summa

$$\int_a^b f(x)\,dx$$

$$\int f(u)\,du$$

set $u = g(x)$ $\quad \dfrac{du}{dx} = g'(x)$

$$du = g'(x)\,dx$$

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$$\int f(u)\,du \rightarrow \int f(g(x))\,g'(x)\,dx$$

$\int 2xe^{x^2}dx$ Let $u = x^2$ $du = 2x\,dx$

$\int e^{x^2}dx$ $\int e^u du = e^u$

$\int \frac{dx}{\ln x}$ Ans. $e^{x^2} + C$