X6F56

early 11-5 (1)

Broadcast media
Human memory
} → learning curve model

Disease spread
Word-of-mouth info
} → logistic model

Ex 1) A large city is under a hurricane watch beginning @ 8:00 am. This warning is broadcast via radio/TV. By 10:00 am 10% of the population has heard the warning. About when will 50% have heard it?

$$P(t) = \underset{\uparrow}{M}(1 - e^{-kt})$$

100

$e^{-2k} = .90$

take ln $-2k = -.1053 \Rightarrow k = +.0527$

Eqn for all times:

$P(t) = 100(1 - e^{-.0527t})$

$50 = 100(1 - e^{-.0527t})$

$.5 = 1 - e^{-.0527t}$

$.5 = e^{-.0527t}$

take ln $-.69315 = -.0527t$

So $t = 13.2$ (hours after 8:00 am)

So about 9pm half population knows.

An actor can learn 35% of a script in 3 weeks. When will he have mastered 80% of the script.

(3)

$P(t) = 100(1 - e^{-kt})$

$P(3) = 100(1 - e^{-k(3)}) = 35$

or $.35 = 1 - e^{-3k}$

$.65 = e^{-3k}$

take ln $-.4307 = -3k$

$k = .1436$

$P(t) = 100(1 - e^{-.1436t})$

$80 = 100(1 - e^{-.1436t})$

$.8 = 1 - e^{-.1436t}$

$.2 = e^{-.1436t}$

take ln's $-1.609 = -.1436t$

so $t = 11.2$ weeks

Ex: @ $t=0$ 2 people start rumor that is spread in a class of 50 students. After 15 minutes 10 students know it. When the class is over after 75 min. How many students know?

Logistic Eqn. $P(t) = \frac{M}{1+Be^{-kMt}}$

Use two constraints:

$P(0) = 2$
$P(15) = 10$

Evaluate B: $2 = \frac{50}{1+B(1)}$

$2(1+B) = 50 \Rightarrow B = 24$

$P(t) = \frac{50}{1+24e^{-k50t}}$

(5)

$$P(t) = \frac{50}{1 + 24e^{-50kt}}$$

$$P(15) = 10 = \frac{50}{1 + 24e^{-750k}}$$

$$10(1 + 24e^{-750k}) = 50$$

$$10 + 240e^{-750k} = 50$$

$$240e^{-750k} = 40$$

$$e^{-750k} = \underline{0.167}$$

$$-750k = -1.789$$

So $k = .00385$

$$P(t) = \frac{50}{1 + 24e^{-1.789t}}$$

$$P(75) = \frac{50}{1 + 24e^{-89.45}}$$

$F = ma$

a = acceleration, it equals $\frac{d(\text{velocity})}{d(\text{time})}$

$a = \frac{dv}{dt}$

Resistance eqn (for fluids) $m\frac{dv}{dt} = -kv \quad (k>0)$

$$\frac{dv}{dt} = -\frac{k}{m}v$$

$$\int \frac{dv}{v} = \int -\frac{k}{m}dt$$

$$\ln v = -\frac{k}{m}t + C \Rightarrow v(t) = e^{-\frac{k}{m}t + C}$$

initial velocity @ $t=0$ - call it v_0

$v(t) = v_0 e^{-\frac{kt}{m}}$ (resistance eqn)

$$v(t) = \frac{ds}{dt}$$

$$\frac{ds}{dt} = V_0 e^{-\frac{kt}{m}} \qquad e^{g(x)}$$

$$s(t) = V_0 \int e^{-\frac{kt}{m}} dt = \frac{V_0 e^{-\frac{kt}{m}}}{\frac{k}{m}} = \rangle$$

$$\boxed{s(t) = \frac{V_0 m}{k} e^{-\frac{kt}{m}}}$$

Prob: Suppose an Iowa class battleship has mass = 51,000 metric tons = 5.1×10^7 kg. and seawater has a $k = 59{,}000$ kg/sec

Assume ship loses power when moving @ 9m/sec

(8)

(a) How far will it coast to a stop?

$$s(t) = \frac{9 \cdot 5.1\times10^{7}}{5.9\times10^{4}} \cdot \frac{\frac{m}{sec} \cdot kg}{\frac{kg}{sec}}$$

★ $$s(t) = 7.78\times10^{3}\, e^{-1.15\times10^{-3}t}$$

↑ how far ship goes

$$\frac{k}{m} = \frac{5.9\times10^{4}}{5.1\times10^{7}} \quad 1.15\times10^{-3}$$

$$v(t) = V_0 - V_0\left(e^{-\frac{k}{m}t}\right)$$

$$= V_0\left(1 - e^{-\frac{k}{m}t}\right)$$

we want $v(t) = 0$

Distance coasted is $\frac{V_0 m}{k}$

so... $$\frac{9\left[\frac{m}{s}\right] \cdot 5.1\times10^{7}[kg]}{5.9\times10^{4}\left[\frac{kg}{s}\right]} = 7.78\times10^{3} \text{ meters}$$

192 lb skater - how long to coast to a stop from 7.5 mph ? $k = .333$

$$\frac{v_0 m}{k} = \frac{11.6\ [\text{slugs}]}{.333} = 198\ [\text{ft}]$$

Infinite Sequence

$a_1, a_2, a_3, \cdots a_n \cdots$

$a_{n+1} = \frac{1}{2} a_n$

$1, 1, 1, \cdots$ $\quad a_n = 1 \ \forall n \in \mathbb{N}$

$a_n = 2^n \quad a_0 = 1$

$a_n = (\sqrt{3})^n + 1$

$a_0 = 0^2 - 1$

$a_1 = 1^2 - 1 = 0$

$a_2 = 2^2 - 1 = 3$

$\vdots$

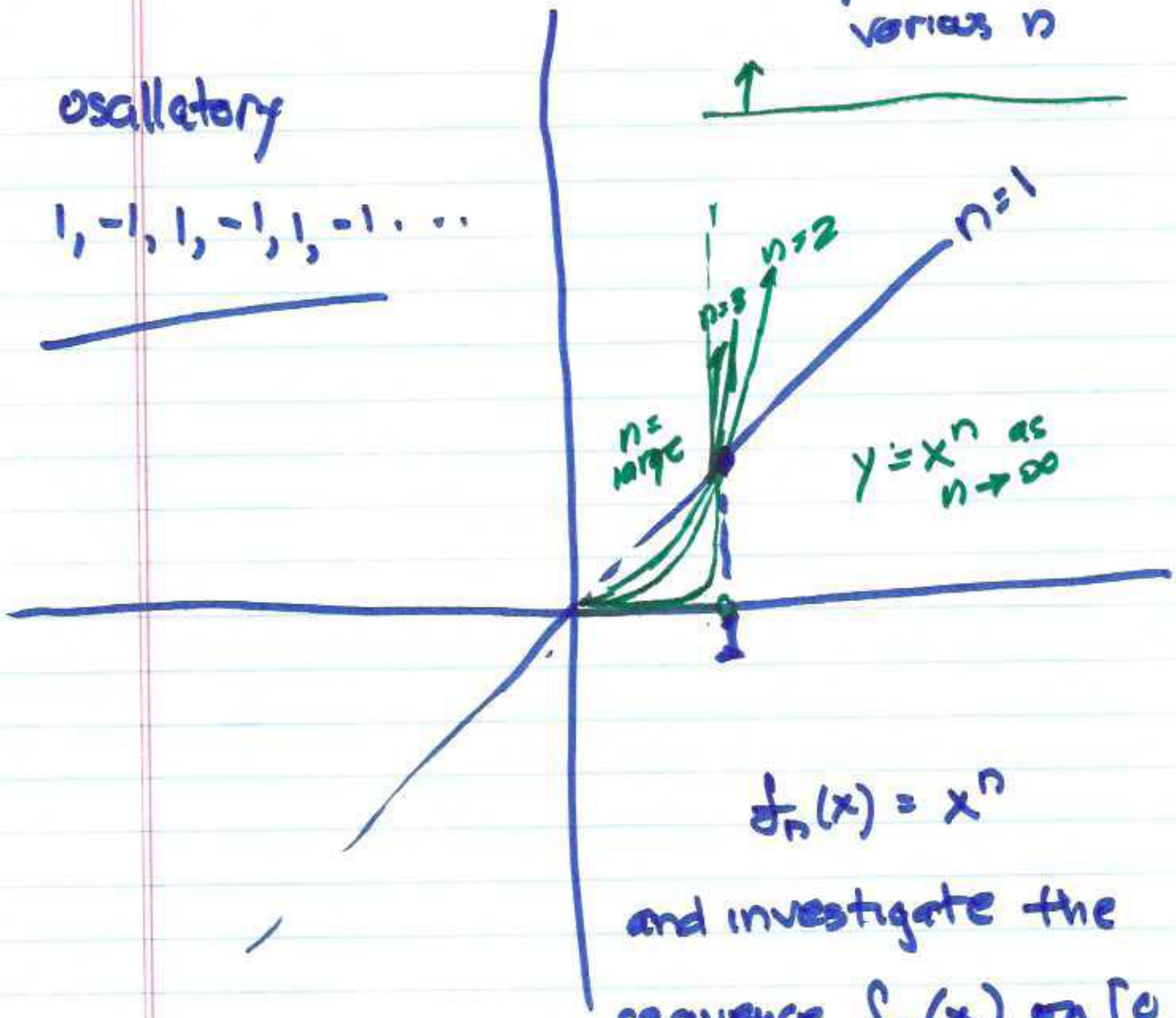
$a_n = n^2 - 1 = n^2 - 1$

10011110_x

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^n}{n!} + \cdots$

$y = x^n$ for various n



oscillatory

1, -1, 1, -1, 1, -1 . . .

$f_n(x) = x^n$

and investigate the sequence $f_n(x)$ on $[0, 1]$

