5YZ9E

early/late 11/3

Some common diff EQ's:

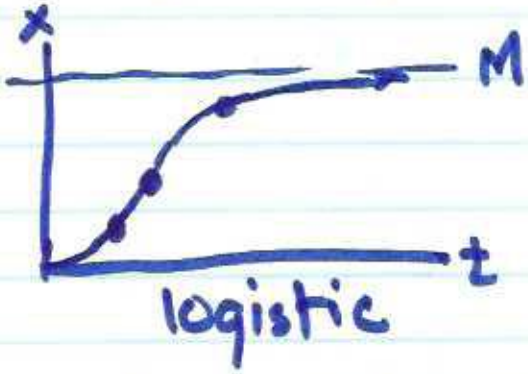
1) $\frac{dx}{dt} = kx$



2) $\frac{dx}{dt} = k(M-x)$



3) $\frac{dx}{dt} = kx(M-x)$



demographer

Solution for #2:

$$\frac{dx}{M-x} = k\,dt \xrightarrow{\int} \int \frac{-du}{u} = -\ln|u| =$$

$$-\ln|M-x| = -\ln(M-x) = kt+C$$

$$\ln(M-x) = -kt-C$$

Differentation w/r/t a Parameter

(Feynman's Method)

First: When a function is a function of two or more variables $f(x,y)$ & you want to differentiate w/r/t one of the say x, you use a partial derivative

∂ $\frac{\partial f(x,y)}{\partial x}$

$$\int \frac{dx}{(1+x^2)^2}$$

Rewrite as $\int \frac{dx}{(a^2+x^2)^2}$

We know $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$

Leibniz' Rule $\left(\frac{\partial}{\partial a}\right) \neq \frac{d}{da}$

$$\frac{d}{da}\int \frac{dx}{a^2+x^2} = \int \frac{\partial}{\partial a}\left(\frac{1}{a^2+x^2}\right)dx =$$

$$\frac{\partial}{\partial a}\left[\frac{1}{a}\arctan\left(\frac{x}{a}\right)\right]$$

$$\int \frac{-2a}{(a^2+x^2)^2}dx = \left[+\frac{1}{a^2}\cdot\arctan\left(\frac{x}{a}\right) + \frac{1}{a}\left(\frac{1}{\left(\frac{x}{a}\right)^2+1}\right)\cdot\left(+\frac{x}{a^2}\right)\right]$$

$$+\int \frac{2a\,dx}{(a^2+x^2)^2} = \frac{1}{a^2}\arctan\left(\frac{x}{a}\right) + \frac{1}{a^3}\cdot\frac{x}{\left(\frac{x}{a}\right)^2+1}$$

$$2\int \frac{dx}{(1+x^2)^2} = \arctan x + \frac{x}{1+x^2} + C$$ ✓

$$M-x = e^{-kt+c} = e^{-kt}\cdot e^{c}$$

Set $x=0$ $\quad M = e^{c}$

Now we know $M-x = Me^{-kt}$

$$M - Me^{-kt} = x \Rightarrow M(1-e^{-kt}) = x(t)$$

So $x(t) = M(1-e^{-kt})$ Learning Curve

#3

$$\frac{dx}{dt} = kx(M-x)$$

$$\frac{dx}{x(M-x)} = kdt \Rightarrow \int \frac{dx}{x(M-x)} = kt+c$$

PFD: $\frac{1}{x(M-x)} = \frac{A}{x} + \frac{B}{M-x}$

$1 = A(M-x) + Bx$ | $1 = BM$

$1 = AM$, so $A = \frac{1}{M}$ | so $B = \frac{1}{M}$

$$\frac{1}{M}\int \frac{dx}{x} + \frac{1}{M}\int \frac{dx}{M-x}$$

$$\downarrow$$

$$\frac{1}{M}\ln|x| + \frac{1}{M}(-\ln(M-x)) = kt+C$$

$$\frac{1}{M}\ln\left[\frac{x}{M-x}\right] = kt+C$$

$$\ln\left(\frac{x}{M-x}\right) = Mkt + C'$$

$$\frac{x}{M-x} = e^{Mkt}\cdot e^{C'} \qquad \underline{e^{C'} = B}$$

Key $x=6$

$$x = (M-x)\cdot\left[e^{Mkt}\cdot B\right]$$

$$x = Me^{Mkt}\cdot B - xe^{Mkt}\cdot B$$

$$x + xe^{Mkt}\cdot B = Me^{Mkt}\cdot B$$

④

$$x\left[1+e^{Mkt\cdot B}\right] = Me^{Mkt\cdot B}$$

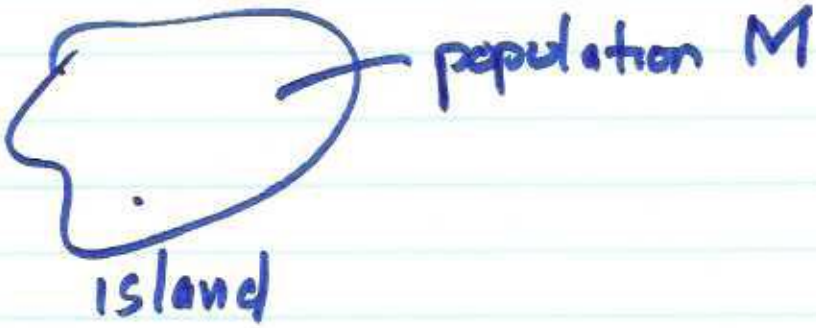
$$\text{so... } x = \frac{Me^{Mkt\cdot B}}{1+e^{Mkt\cdot B}}$$

$$x = \frac{M}{\frac{1}{B}e^{-Mkt} + 1}$$

verhulst's equation

$$x = \frac{M}{1 + B'e^{-Mkt}}$$

need 2 data points



Let $P(t)$ = # people sick @ time t

so... $M - P(t)$ people are not infected

$$\Delta P = k P(t)(M - P(t))\,\Delta t$$

$$\lim_{\Delta t \to 0} \frac{\Delta P}{\Delta t} = P'(t) = kP(t)(M - P(t))$$

