

E9CVP

early
10/8
①

8.2 Integration by Parts (Partial Integration)

Start with two functions: $u(x) \div v(x)$

$$\frac{d}{dx}(u(x) \cdot v(x)) \stackrel{?}{=} u'(x)v(x) + u(x)v'(x)$$

$$\int \frac{d}{dx}(u(x)v(x)) dx = \int u'(x)v(x) dx + \int v'(x)u(x) dx$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ u'(x)dx & & \\ \downarrow & & \\ du & & \end{array} \quad \begin{array}{c} uv \\ = \int v du + \int u dv \end{array}$$

$$\star uv = \int v du + \int u dv$$

no brain easy difficult

① $\int u dv = uv - \int v du$

②

$$\int x e^x dx$$

$u = x$	$du = 1 dx$
$v = e^x$	$dv = e^x dx$

$$u =$$

$$v = \int x e^x dx = x e^x - \int (1) e^x dx$$

$$\int x e^x dx = x e^x - \int e^x dx$$

$$(x e^x \cdot e^x) = e^x (x - 1)$$

③

$$\text{Ex 1: } \int \underset{\downarrow}{x} \cos x dx$$

$$u = x \quad v = \sin x$$

$$du = dx \quad dv = \cos x dx$$

Base Formula: $\int u dv = uv - \int v du$

$$\int \underset{\uparrow}{x} \cos x dx = \underset{\uparrow}{x} (\sin x) - \int \sin x dx$$

$u \quad dv$

$$\int x \cos x dx = x \sin x + \cos x$$

$$\int u dv \quad (4)$$

$$- \int \underbrace{x}_{\downarrow u} \underbrace{\cos x}_{\uparrow \frac{dv}{dx}} dx = x \sin x - \int \sin x dx$$

$$= x \sin x + \cos x$$

$$\int x e^x dx = x e^x - \int (1) e^x dx$$

$x = u$
 $v = \int e^x dx = e^x$

$\downarrow$
 $x e^x - e^x = \underline{e^x (x - 1)}$

$$\int \underbrace{e^x}_{\downarrow u} \underbrace{\cos x}_{\frac{dv}{dx}} dx$$

$$u = e^x$$

$$v = \sin x$$

$$du = e^x dx$$

$$dv = \cos x dx$$

$$\int u dv = uv - \int v du \quad (5)$$

1st
pass

$$\int e^x \cos x dx = e^x \sin x - \int \sin x \cdot e^x dx$$

$$\int \underbrace{e^x}_{u} \underbrace{\sin x dx}_{dv}$$

$$u = e^x$$

$$v = -\cos x$$

$$du = e^x dx$$

$$dv = \sin x dx$$

$$\int e^x \sin x dx = -e^x \cos x - \int (-\cos x) e^x dx + \int e^x \cos x dx$$

$$\int e^x \cos x dx = e^x \sin x - \left[-e^x \cos x + \int e^x \cos x dx \right]$$

$$2 \int e^x \cos x dx = e^x \sin x + e^x \cos x$$

⑥

$$\int e^x \cos x dx = \frac{1}{2} \left[e^x (\sin x + \cos x) \right]$$

$$\int \underline{\underline{x^3 e^x dx}}$$

Tabular Integration

$\frac{d}{dx}$	$\int dx$
x^3	e^x
$3x^2$	e^x
$6x$	e^x
6	e^x
0	e^x

parity $\pm$

$\swarrow$?

$$x^3 e^x + 3x^2 e^x + 6x e^x - 6e^x$$

⑦

$$\int x^3 e^x dx = e^x [x^3 - 3x^2 + 6x - 6]$$

③ $\int t^2 \cos t dt$

$\frac{d}{dt}$	$\int dt$
t^2	$\cos t$
$2t$	$\sin t$
2	$-\cos t$
0	$-\sin t$

$$\int t^2 \cos t dt = t^2 \sin t + 2t \cos t - 2 \sin t \quad \checkmark$$

⑧

⑥ $\int_1^e x^3 \ln x \, dx$

d/dx	$\int dx$
x^3	$\ln x$
$3x^2$	$\underline{x \ln x - x}$ ←
$6x$	$?$
6	
0	
$\approx$	

d/dx	$\int dx$
$\ln x$	x^3
$1/x$	$\frac{x^4}{4}$
$-\frac{1}{x^2}$	$\frac{x^5}{20}$
$\frac{2}{x^3}$	$\vdots$

$$\int u dv = uv - \int v du$$

⑨

$$\int \ln x dx = \int \overset{x}{\underset{\downarrow \frac{1}{x}}{1}} \ln x dx$$

$$x \ln x - \int x \cdot \frac{1}{x} dx$$

$$\int dx = x$$

$$\int \ln x dx = x \ln x - x$$

$$\int \overset{x^2}{\underset{\downarrow}{x}} \ln x dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$$

$$\frac{x^2}{2} \ln x - \frac{1}{2} \int \frac{1}{x} dx$$

$$\frac{x^2}{2} \ln x - \frac{1}{2} \ln x$$

$$\frac{\ln x}{2} (x^2 - 1)$$

(10)

$$(13) \int \underset{\downarrow}{x} \overset{\uparrow}{\sec^2 x} dx = x \tan x - \int (1) \tan x dx$$

$$\int \frac{\sin x}{\cos x} dx = \int \frac{-d(\cos x)}{\cos x} = -\ln |\cos x|$$

$$\int x \sec^2 x dx = x \tan x + \ln |\cos x|$$

$$(11) \int \overset{\uparrow}{1} \underset{\downarrow}{\arctan y} dy$$
$$\frac{1}{1+y^2}$$

$$\underline{\int \frac{dx}{1+x^2} = \arctan x}$$

$$\int \arctan y dy = y \arctan y - \int \frac{y}{1+y^2} dy$$

(11)

$$u = 1+y^2 \quad du = 2y dy \text{ so } dy = \frac{du}{2y}$$

$$\int \frac{y}{1+y^2} dy = \int \frac{(du/2)}{u} = \frac{1}{2} \int \frac{du}{u} =$$

$$\frac{1}{2} \ln |u| = \frac{1}{2} \ln |1+y^2|$$

$$\int \arctan y dy = y \arctan y - \ln |\sqrt{1+y^2}|$$

(45) $\int \cos \sqrt{x} dx$ Let $u = \sqrt{x}$
 $du = \frac{dx}{2\sqrt{x}}$

$$\int \cos u \cdot (2\sqrt{x}) du$$

↑
u

$$2 \int u \cos u du$$

(12)

$\frac{d}{du}$	$\int du$
u	$\cos u$
1	$\sin u$
0	$-\cos u$

 $\Rightarrow u \sin u + \cos u$

$$\int \cos \sqrt{x} dx = 2 \int \overset{\sqrt{x}}{u} \cos u du = 2 [u \sin u + \cos u]$$

$$= 2 [\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}]$$

46) $\int \sqrt{x} \cdot e^{\sqrt{x}} dx$ let $u = \sqrt{x}$

$$\int u e^u (2u du)$$

$$\frac{du}{du} = \frac{dx}{2u}$$

(13)

$$2 \int u^2 e^u du$$

d/du	$\int du$
u^2	e^u
$2u$	e^u
2	e^u
0	e^u

$$e^u u^2 - 2u e^u + 2e^u = e^u [u^2 - 2u + 2]$$

$$2e^u \int u^2 - 2u + 2 \xrightarrow{u=\sqrt{x}} 2e^{\sqrt{x}} [x - 2\sqrt{x} + 2]$$

(14)

8.3 Trig Integral "tricks"

① $\int \sin^m x \cos^n x dx$

Case A: m is odd, ... then $m = 2k+1$

then express $\frac{m}{2}$ copies of $\sin^2 x = (1 - \cos^2 x)$

Ex: $\int \sin^3 x \cos^2 x dx = \int \sin^2 x \cos^2 x \underbrace{(\sin x dx)}_{d(-\cos x)}$

$$= \int (1 - \cos^2 x) \cos^2 x d(-\cos x)$$

$$= -\int (\cos^2 x - \cos^4 x) d(\cos x)$$

if $u = \cos x$

$$\int u^2 - u^4 du$$

$$\boxed{-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5}}$$

(15)

$$(2) \int \sin^m x \cos^n x dx$$

Case B: n is odd

Reserve 1 factor of $\cos x$ to form

$d(\sin x) = \cos x dx$, then convert

even power of cosines to sines

$$\text{Ex: } \int \cos^5 x dx = \int \cos^4 x \overset{\downarrow \cos x dx}{d(\sin x)}$$

$$= \int (1 - \sin^2 x)^2 d(\sin x)$$

$$= \int (1 - 2\sin^2 x + \sin^4 x) d(\sin x)$$

$$= \boxed{\sin x - 2 \frac{\sin^3 x}{3} + \frac{\sin^5 x}{5}}$$

(16)

$$\int \sin^m x \cos^n x dx \quad m, n \text{ both even}$$

$$\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}}$$

$$\sin x = \sqrt{\frac{1 - \cos 2x}{2}}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\text{Ex: } \int \tan^4 x dx = \int \tan^2 x \cdot \tan^2 x dx$$

$$\tan^2 x = \sec^2 x - 1$$

$$\int \tan^2 x \cdot (\sec^2 x - 1) dx$$

$$= \int \tan^2 x \sec^2 x dx - \int \tan^2 x dx$$
$$\frac{\tan^3 x}{3} - \tan x - x$$