

w/5
early
①

Integration:

8.1 - Use table

Ex 1. $\int_3^5 \frac{2x-3}{\sqrt{x^2-3x+1}} dx$

Let $x^2 - 3x + 1 = u$

then $\frac{du}{dx} = 2x - 3$ so $du = (2x - 3) dx$

$\int_3^5 \frac{1}{\sqrt{u}} du = 2\sqrt{u} \Big|_{x=3}^5$ $u^{-\frac{1}{2}} \xrightarrow{\int} 2\sqrt{u}$

$\left[2\sqrt{x^2 - 3x + 1} \right]_3^5 \sim \underline{4.63}$

(2)

$$\text{Ex 2} \quad \int \frac{dx}{\sqrt{8x-x^2}} = \int \frac{dx}{\sqrt{-(x^2-8x+16)+16}}$$

$$= \int \frac{dx}{\sqrt{16-(x-4)^2}}$$

↗ ↘
a=4

Form

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right) + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} \quad \text{where } a=4$$

$$\begin{aligned} u &= x-4 \\ du &= dx \end{aligned}$$

$$\frac{\arcsin\left(\frac{u}{4}\right) + C}{\arcsin\left(\frac{x-4}{4}\right) + C}$$

(3)

$$\text{Ex 3 - } \int (\cos x \sin 2x + \sin x \cos 2x) dx$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \\ 2x & x & 2x & x \end{matrix}$

$$\sin(2x + x) = \sin(3x)$$

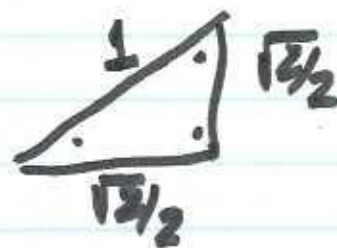
$$\int \sin(3x) dx = \boxed{-\frac{\cos 3x}{3} + C}$$

$$\text{Ex 4 - } \int_0^{\pi/4} \frac{dx}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x}$$

$$= \int_0^{\pi/4} \frac{(1 + \sin x) dx}{1 - \sin^2 x} = \int_0^{\pi/4} \frac{1 + \sin x}{\cos^2 x} dx$$

$$\begin{aligned}
 &= \int_0^{\pi/4} \frac{dx}{\cos^2 x} + \int_0^{\pi/4} \frac{\sin x dx}{\cos^2 x} \\
 &= \int_0^{\pi/4} \sec^2 x dx + \int_0^{\pi/4} \tan x \sec x dx
 \end{aligned}$$

$$\left[\tan x \right]_0^{\pi/4} + \left[\sec x \right]_0^{\pi/4}$$



$$(1 - 0) + \left(\frac{\sqrt{2}}{2} - 1 \right) = \frac{\sqrt{2}}{2}$$

Ex 5 $\int \frac{3x^2 - 7}{3x + 2} dx$

rational function
is ratio of 2
poly's

$$\frac{3x^2 - 7}{3x + 2} = x - \frac{2}{3} - \frac{17}{3(3x + 2)}$$

integrate
this!

⑥

Second try Let $y = x^2$ $dy = 2x dx$

$$\text{so } dx = \frac{dy}{2x} \\ = \frac{dy}{2\sqrt{y}}$$

$$\int \frac{y}{1+y} \frac{dy}{2\sqrt{y}}$$

$$\frac{1}{2} \int \frac{\sqrt{y}}{1+y} = ?$$

Third try Let $x = \tan \theta$ $dx = \sec^2 \theta d\theta$

$$\int \frac{\tan^2 \theta}{\sec^2 \theta} \underbrace{\sec^2 \theta d\theta}_{dx} = \int \tan^2 \theta d\theta$$

$$= \int (\sec^2 \theta - 1) d\theta = \int \sec^2 \theta d\theta - \int d\theta$$

$$\boxed{\tan \theta - \theta + C}$$

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$$\textcircled{1A} \int \underline{\csc t} \sin 3t \, dt$$

$$\text{1st try: } \sin 3t = \sin(t + 2t) =$$

$$\sin t \cos 2t + \cos t \sin 2t$$

$$\downarrow$$
$$\cos^2 t - \sin^2 t$$

$$\downarrow$$
$$2 \sin t \cos t$$

$$\sin 3t = \sin t \cos^2 t - \sin^3 t + 2 \sin t \cos^2 t$$

$$\csc t \sin 3t = \cos^2 t - \sin^2 t + 2 \cos^2 t$$

$$= \int (3 \cos^2 t - \sin^2 t) \, dt$$

$$= 3 \int \cos^2 t \, dt - \int \sin^2 t \, dt$$

(8)

$$\sin^2 t + \cos^2 t = 1 \text{ so } \sin^2 t = 1 - \cos^2 t$$

$$\star \cos^2 t = 1 - \sin^2 t$$

$$= 3 \int \cos^2 t dt - \int (1 - \cos^2 t) dt$$

$$= 4 \int (\cos^2 t dt - 1) dt$$

$$34) \int e^{z+e^z} dz$$

$$\text{let } u = z + e^z \mid du = (1 + e^z) dz$$

$$\int \underbrace{e^u}_{\text{}} (1 + e^z) du$$

⑨

Instead try $u = e^z$ then $du = e^z dz$

$$\cancel{e^z} e^{z+e^z} = e^z \cdot e^{e^z} = u e^u$$

$$\int u e^u dz \rightarrow \int u e^u \frac{du}{e^z} = \int e^u du$$

$$= e^u = \cancel{e^z} e^{e^z}$$

$$40) \int \frac{\sqrt{x} dx}{1+x^3} = \int \frac{x^{1/2} dx}{1+x^3} \quad u = x^{3/2}$$

$$du = \frac{3}{2} x^{1/2} dx$$

$$\text{so } dx = \frac{2}{3} \cdot \frac{1}{x^{1/2}} du$$

$$\int \frac{x^{1/2} dx}{1+u^2} \rightarrow \int \frac{x^{1/2} \left(\frac{2}{3} \frac{1}{x^{1/2}} \right) du}{1+u^2}$$

(10)

$$\therefore \frac{2}{3} \int \frac{du}{1+u^2} = \frac{2}{3} \arctan u \quad \curvearrowright$$

$$\frac{2}{3} \arctan(x^{3/2})$$