MKV92

early 10/20 (1)

1st kind $\int_c^\infty f(x)\,dx = \lim_{b\to\infty} \int_c^b f(x)\,dx$

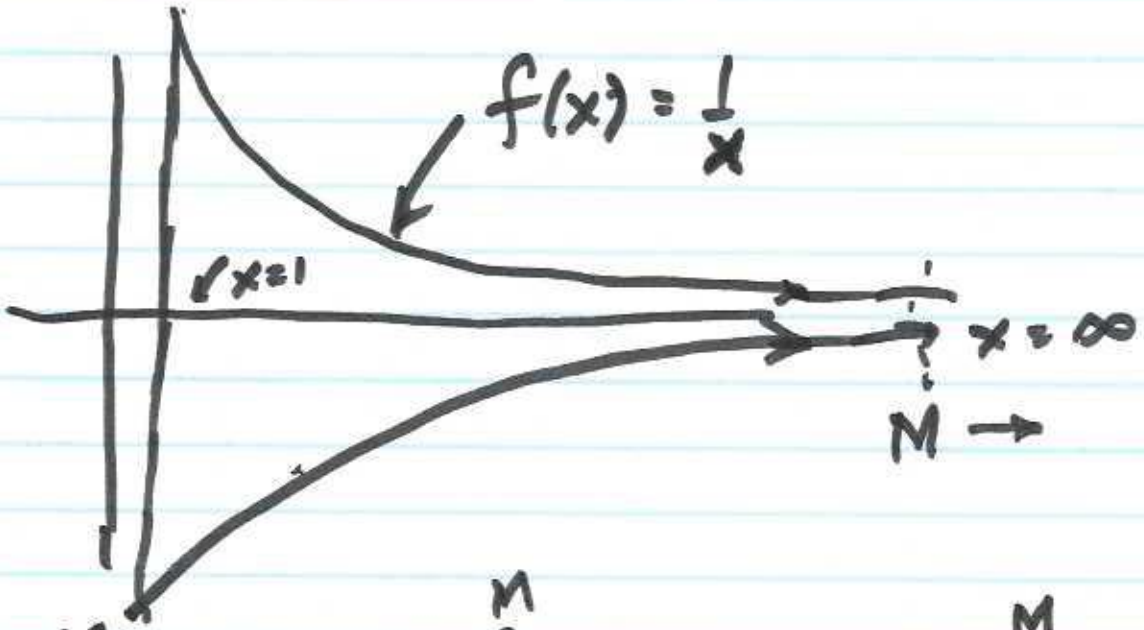
$$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_{-b}^{b} f(x)\,dx$$

2nd kind $$\int_0^1 \frac{dx}{\sqrt{x}} = \int_0^1 x^{-1/2}\,dx = \left[2x^{\frac{1}{2}}\right]_0^1 = 2$$

$$\int_0^1 \frac{dx}{x} = \left[\ln x\right]_0^1 = 0 - (-\infty) = \infty$$

Gabriel's Horn

Consider



$$\text{Vol} = \int_1^M \frac{\pi}{x^2}\,dx = \pi\int_1^M x^{-2}\,dx = \pi\left[-x^{-1}\right]_1^M$$

$$= \pi\left(-\frac{1}{M} - \left(-\frac{1}{1}\right)\right) = \pi\left(1 - \frac{1}{M}\right)$$

$$\lim_{M\to\infty} \pi\left(1 - \frac{1}{M}\right) = \boxed{\pi} \text{ vol of horn}$$

(3) $\overbrace{f(x)}^{ds}$ $ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

Surface : $2\pi \int_M f(x)\sqrt{1+(f'(x))^2}\,dx$

$= 2\pi \int_1^M \left(\frac{1}{x}\right)\sqrt{1+\left(-\frac{1}{x^2}\right)^2}\,dx$

$= 2\pi \int_1^M \frac{\sqrt{1+\frac{1}{x^4}}}{x}\,dx$

$\geq 2\pi \int_1^M \frac{dx}{x} = 2\pi \ln|x|\Big|_1^M$

smaller than $\longrightarrow$ $= 2\pi \ln M$

total surface area

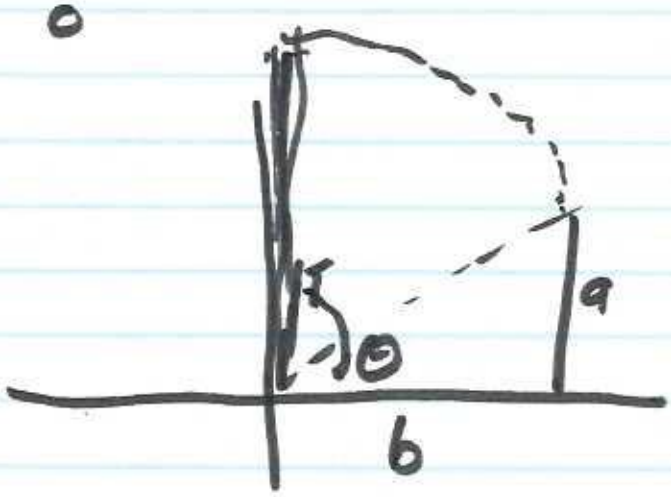
$\lim_{M\to\infty} 2\pi \ln M = \infty$

Container has finite vol (π)

" " infinite area (∞)

④

1) $\int_0^{\infty} \frac{dx}{x^2+1} = [\arctan x]_0^{\infty}$ $\quad \infty \leftarrow 90^\circ$, $\quad 0 \leftarrow 0$



$$\int_0^{\infty} \frac{dx}{x^2+1} = \frac{\pi}{2}$$

5) $\int_{-1}^{1} \frac{dx}{x^{2/3}} = 3\sqrt[3]{x}\,\Big|_{-1}^{1} \quad \frac{1}{3}\left(1-(-1)\right) = \frac{2}{3}$

21) $\int_{-\infty}^{0} \theta e^{\theta} d\theta$

$e^{x}(x-1)\Big|_{-\infty}^{0}$

$= -1 - (\ ?\ )$

$\int x e^{x} dx$

d/dx	∫dx
x	e^x
1	e^x
0	e^x

$xe^{x} - e^{x} =$

$e^{x}(x-1)$

$e^{-\infty}\cdot(-\infty) = [0\cdot\infty]$ L'Hôpital

$\lim_{x\to-\infty} \frac{x-1}{e^{x}} = \lim_{x\to-\infty}\left(\frac{1}{e^{x}}\right) = \frac{1}{0} = \infty$

25) $\int_0^1 x \ln x \, dx = \lim_{\varepsilon \to 0} \int_{\varepsilon}^1 x \ln x \, dx$

d	∫dx
x	$\ln x$
1	$x \ln x - x$
0	

practice

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15) $\int_0^1 \frac{\theta + 1}{\sqrt{\theta^2 + 2\theta}} \, d\theta$

$(\theta^2 + 2\theta + 1) - 1$

$(\theta + 1)^2 - 1$

$u = \theta + 1$

$\theta : 1$

$\int_0^1 \frac{u}{\sqrt{u^2 - 1}} \, du$

(3)

Try $u = \sec x$ $du = \sec x \tan x \, dx$

$\sqrt{\sec^2 x - 1} = \tan x$

$$\int_{\theta=0}^{\theta=1} \frac{\sec x}{\tan x} \cdot \sec x \tan x \, dx$$

$$\int_{\theta=0}^{\theta=1} \sec^2 x \, dx = \tan x$$

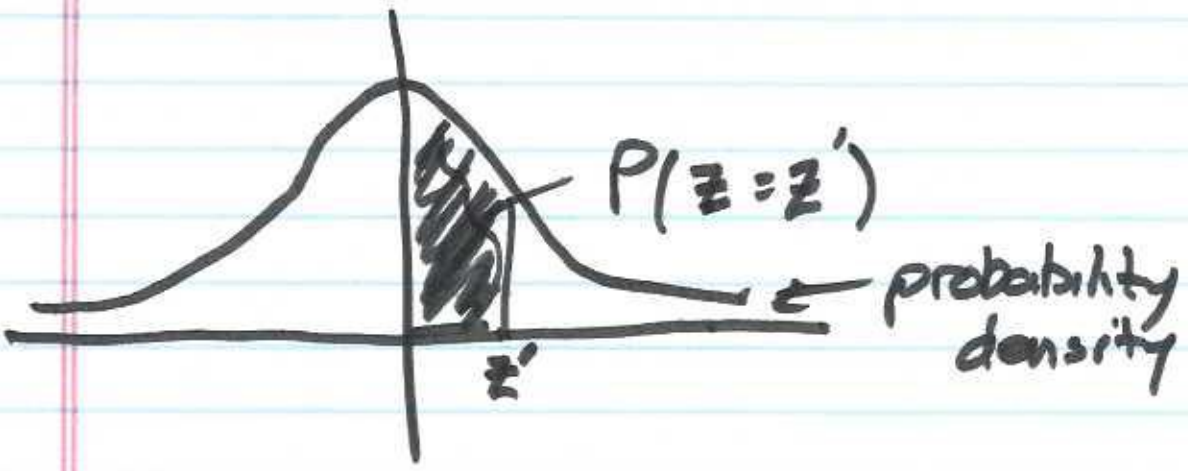
$$\tan(\operatorname{arcsec} u) \Big|_0^1$$

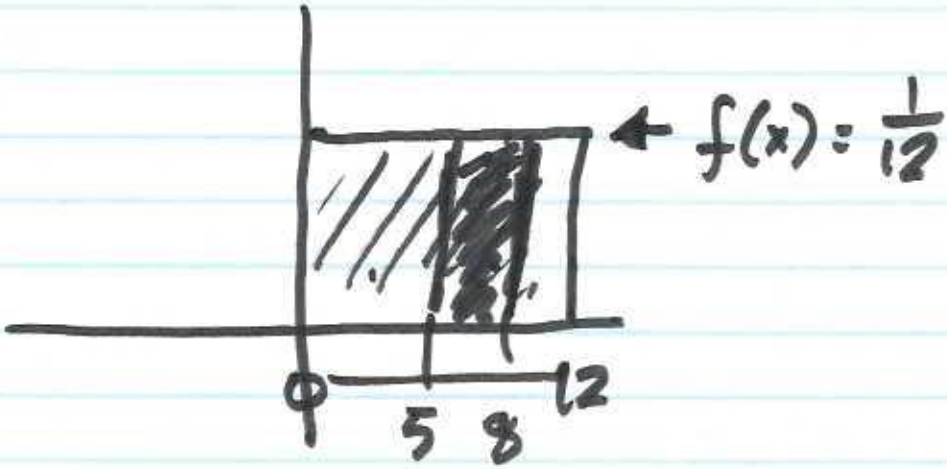
$$= \tan(\operatorname{arcsec}[\theta+1]) \Big|_0$$

$$= \tan(\operatorname{arcsec} 2) \cdot \tan(\operatorname{arcsec} 1)$$

$$Si(x) := \int_0^x \frac{\sin t}{t} dt \leq \int_0^x \frac{1}{t} dt$$

(8)





$3\left(\frac{1}{12}\right) = .25 = 25\%$

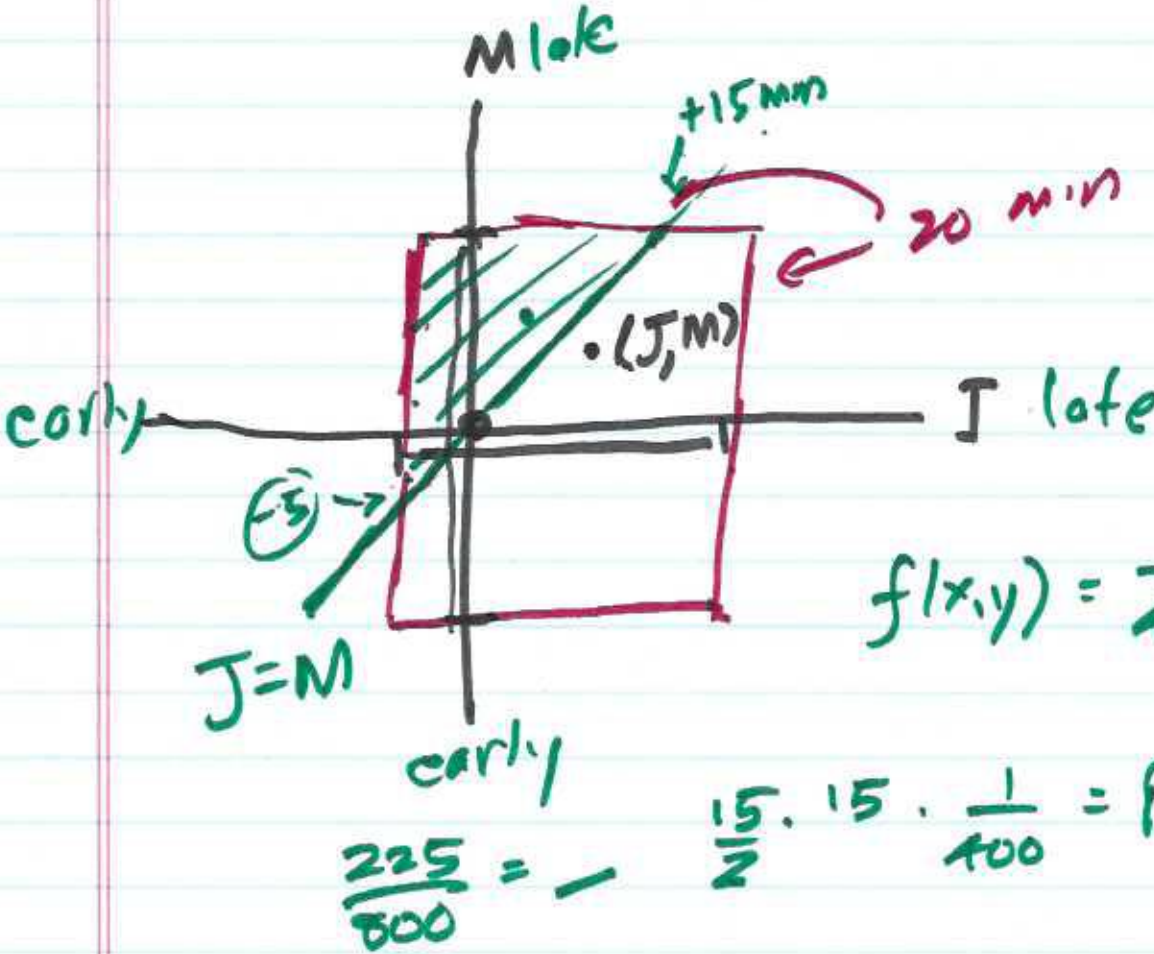
(1)

John and Mary attend a meeting at noon

John is always between 5 min early and 15 min late

Mary, being more responsible is always between 10 min early and 10 min late

What is the probability that John will be earlier to the noon meeting.



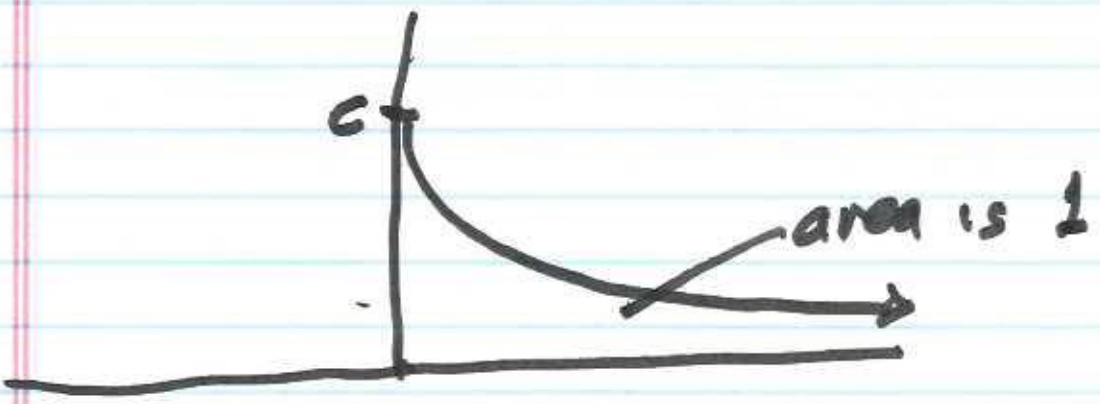
$f(x,y) = \frac{1}{400}$

$\frac{225}{500} = -$

$\frac{15}{2} \cdot 15 \cdot \frac{1}{400} = P(J<M)$

(10)

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ ce^{cx} & \text{if } x \geq 0 \end{cases}$$



What is the mean of the distribution

Def^n: $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

for this dist: $\mu = \int_{0}^{\infty} x(ce^{-cx})\,dx$

$$= c\int_{0}^{\infty} xe^{-cx}\,dx$$

(11)

d/dv	$\int dv$
x	e^{-cx}
1	$\frac{e^{-cx}}{-c}$
0	$\frac{e^{-cx}}{c^2}$

(+)

(-)

$$\left[\frac{xe^{-cx}}{-c} - \frac{e^{-cx}}{c^2}\right]_0^M \qquad M \to \infty$$

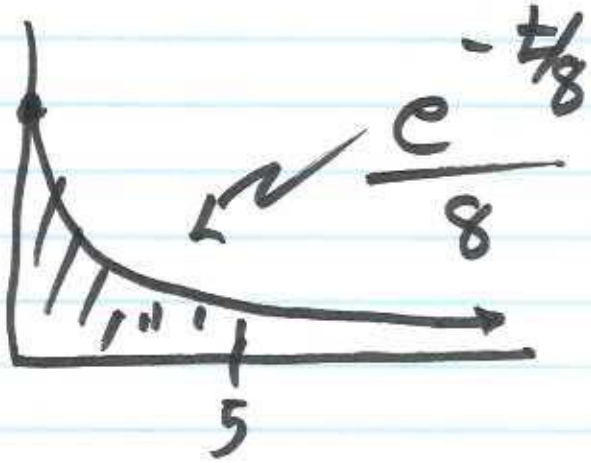
$$\lim_{M\to\infty}\left(\frac{Me^{-cM}}{-c} - \frac{e^{-cM}}{c^2}\right) - \left(0 - \frac{1}{c^2}\right)$$

$$0 - 0 + \frac{1}{c^2}$$

Chip failure dist.

$$f(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{1}{8} e^{-t/8} & \text{if } t \geq 0 \end{cases}$$

What is probability that chip will fail within 5 yrs?



$$P(0 \leq T \leq 5) = \int_0^5 f(t)\,dt =$$

$$\frac{1}{8}\int_0^5 e^{-t/8}\,dt = \frac{1}{8}\left[e^{-t/8}\left(-\frac{1}{8}\right)\right]_0^5 =$$

$$-\frac{1}{64}\left[e^{-t/8}\right]_0^5 = -\frac{1}{64}\left[e^{-5/8}-1\right]$$

$$\frac{1}{64}\left(1-e^{.625}\right) = \underline{0.465}$$

$$\mu = \int_0^\infty t\left(\frac{1}{8}e^{-t/8}\right)dt \qquad \underline{\int x e^x\,dx}$$