

F2JY8

early
10/15
①

Partial Fraction Integrals:

A rational function is a ratio of polynomials. Example:

$$f(x) = \frac{5x-3}{x^2-2x-3}$$

This is difficult to do directly, so we reduce it the following way:

$$\frac{5x-3}{x^2-2x-3} = \frac{\textcircled{8} \textcircled{2}}{x+1} + \frac{3 \textcircled{1}}{x-3}$$

Integrating terms on right is easy (you get logs).

We want a systematic way of doing such reductions.

(2)

$$\frac{5x-3}{x^2-2x-3} = \frac{A}{x-3} + \frac{B}{x+1}$$

factor $(x-3)(x+1)$

$$5x-3 = A(x+1) + B(x-3)$$

No trick $5x = Ax + Bx \Rightarrow A+B=5$

$$-3 = A-3B$$

Trick: try $x=-1$: ~~$5(-1) = 0 + (-4B)$~~

~~$-5 = -4B \Rightarrow B = \frac{5}{4}$~~

$$5(-1)-3 = 0 + B(x-3)$$

$$-8 = -4B \Rightarrow B=2$$

~~$B = \frac{5}{4}$
 $A = \frac{3}{4}$~~

③

$$\int \frac{(5x-3)dx}{x^2-2x-3} = \int \frac{2dx}{x+1} + \int \frac{3}{x-3}$$

↓

↓

$$\boxed{2 \ln(x+1) + 3 \ln(x-3)}$$

$$\boxed{x > 3}$$

Always first determine degree of num/den

②② $\int \frac{3t^2 + t + 4}{t^2 + t} dt$

$$t(t+1)$$

$$3 + \frac{-2t+4}{t^2+t}$$

First $t^2 + t \left| \begin{array}{r} 3t^2 + t + 4 \\ 3t^2 + 3t \\ \hline -2t + 4 \end{array} \right.$

④

$$\int \frac{3t^2 + t + 4}{t^2 + t} dt = \int 3 dt - \int \frac{2t - 4}{t(t+1)}$$

$$\frac{2t - 4}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$$

$$2t - 4 = A(t+1) + Bt$$

$$t=0 \quad -4 = A(1) \Rightarrow A = -4$$

$$t=-1 \quad -6 = (-4)(0) + (-B) \Rightarrow B = 6$$

$$\frac{2t - 4}{t(t+1)} = \frac{-4}{t} + \frac{6}{t+1}$$

$$\frac{-4(t+1) + 6t}{t(t+1)}$$

$$\frac{\frac{a}{b} + \frac{c}{d}}{\frac{ad+bc}{bd}}$$

(5)

$$\textcircled{22} \int \frac{3t^2 + t + 4}{t^2 + t} dt = \int 3 dt - \int \frac{4}{t} dt + \int \frac{6}{t+1} dt$$
$$= 3t - 4 \ln t + 6 \ln(t+1)$$

Quadratics present in denom.

$$\int \frac{-2x+4}{(x^2+1)(x-1)^2} =$$

↑

$$\frac{-2x+4}{(x^2+1)(x-1)^2} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$= (Ax+B)(x-1)^2 + C \overbrace{(x^2+1)(x-1)}^{x^3 - x^2 + x - 1} + D(x^2+1)$$

$$Ax(x^2-2x+1) + B(x^2-2x+1) + C(x^3-x^2+x-1) \Bigg\}$$

$Dx^2 + D$

$$\begin{aligned}
 & \overset{0}{\underbrace{}} \quad \overset{0}{\underbrace{}} \\
 & x^3(A+C) + x^2(-2A+B-C+D) + \\
 & x(A-2B+C) + 1(B-C+D) = -2x+4
 \end{aligned}$$

Know: ① $A = -C$

② $B+D+C = 0$

③ $x(-2B) = -2x \rightarrow B=1$

④ $1(B-C+D) = 4$

Add ② : ④ $2+2D = 4 \rightarrow D=1$

$C = -2$

$A = 2$

$$\int \frac{2x+1}{x^2+1} dx = \int \frac{-2}{x-1} dx + \int \frac{dx}{(x-1)^2}$$

⑦

$$\int \frac{(-2x+4)dx}{(x^2+1)(x-1)^2} = \int \frac{(2x+1)dx}{x^2+1} - 2 \int \frac{dx}{x-1} + \int \frac{dx}{(x-1)^2}$$

$$= 2 \int \frac{x dx}{x^2+1} + \int \frac{dx}{x^2+1} - 2 \int \frac{dx}{x-1} + \int \frac{dx}{(x-1)^2}$$

Let $u = x^2+1$
 $du = 2x$

$\downarrow$

$\ln(x^2+1)$

$\downarrow$

$\arctan x$

$\downarrow$

$\ln(x-1)$

$\downarrow$

$\frac{-1}{x-1} + C$

$$\int \frac{(x-1)dx}{(x+1)^3} = \int \frac{A}{x+1} + \int \frac{B}{(x+1)^2} + \int \frac{C}{(x+1)^3}$$

$$x-1 = A(x+1)^2 + B(x+1) + C$$

⑧

$$x-1 = A(x^2+2x+1) + B(x+1) + C$$

$$\underline{x-1} = x^2(A) + x(2A+B) + 1(A+B+C)$$

1) $A=0$

2) $B=1$

3) $C=-2$

$$\int \frac{(x-1)dx}{(x+1)^3} = \int \frac{1 dx}{(x+1)^2} \quad \begin{array}{l} u = x+1 \\ du = dx \end{array}$$

↓

$$\int \frac{du}{u^2} = -\frac{1}{u}$$

→ $\left(-\frac{1}{x+1} - \frac{1}{(x+1)^2} \right) - 2 \int \frac{dx}{(x+1)^3}$

u^{-3}

⑨

(26) $\int \frac{s^4 + 81}{s(s^2 + 9)^2} = ?$

$$\frac{s^4 + 81}{s(s^2 + 9)^2} = \frac{A}{s} + \frac{Bs + C}{s^2 + 9} + \frac{Ds + E}{(s^2 + 9)^2}$$

$$s^4 + 81 = A(s^2 + 9)^2 + (Bs + C)s(s^2 + 9) + s(Ds + E)$$

$$\checkmark = A[s^4 + 18s^2 + 81] + [Bs^4 + 9Bs^2 + Cs^3 + 9sC] + Ds^2 + Es$$

$$s^4 + 81 = s^4(A + B) + s^3(C) + s^2(18A + 9B + D) + s(9C + E) + 1(81A)$$

$$A + B = 1 \quad (A = 1) \quad (B = 0)$$

(10)

$$C=0 \quad E=0$$

$$18+0+D=0 \Rightarrow D=-18$$

$$\int \frac{(s^4+81)ds}{s(s^2+9)^2} = \int \frac{ds}{s} + \int \frac{-18s ds}{(s^2+9)^2}$$

$$\downarrow$$

$$\ln|s|$$

$$\downarrow$$

$$u=s^2+9$$

$$du=2s$$

$$\ln|s| - \frac{9}{s^2+9} + C$$

$$-9 \int \frac{2s ds}{(s^2+9)^2} = -9 \int \frac{du}{u^2}$$

$$\downarrow$$

$$\frac{-9}{u} \rightsquigarrow -\frac{9}{s^2+9}$$