There is no set of all sets.

Pf: FSOC Let S = set of <u>all</u> sets.

Let M = set of all sets not members of themselves

$$M = \{A \in S : A \notin A\}$$

Consider: $M \in M \Rightarrow M \notin M \Rightarrow\Leftarrow$

Must be the case $M \notin M \Rightarrow \Rightarrow\Leftarrow$

$ZF \rightarrow ZFC$

<u>NBG</u>

$\text{class} \rightarrow MK$

$$\prod_{\alpha \in \Delta} S_\alpha \neq \phi$$

$\aleph_0$ 1^{st} infinite ordinal

$$0, 1, 2, 3 \cdots \mid \omega, \omega+1, \omega+2, \cdots, \cdots \Omega \le \aleph_1$$

limit ordinals

regular ordinals

1^{st} uncountable ordinal

class of ordinals ON

Burali-Forti Paradox

$1+\omega = \omega$

$\omega+1 \neq \omega$

${}^*\mathbb{R} > \mathbb{R}$

$(1, \frac{1}{2}, \frac{1}{3}, \cdots \to 0$ $\quad 0$

$(1, \frac{1}{4}, \frac{1}{9}, \cdots \to 0$ $\quad 0$

$\to (0, 0, 0, \cdots)$

Thm card$(2^A) >$ card A

Pf: Certainly card $A \leq$ card 2^A

FSOC Suppose card $A =$ card 2^A

$\exists \phi : A \to 2^A$ which is bijective

Let $a \in A$, if $a \in \phi(a) \Rightarrow a$ is "good"

$a \notin \phi(a) \Rightarrow a$ is "bad".

Consider $B = \{$bad elements$\}$

$\phi(b) = B$

Is b good or bad?

<u>b not good</u> / b is not bad $\Rightarrow\!\Leftarrow$

Cardinality & Ordinality

$\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\} \longrightarrow$

0 1 2 3 4

②

card$(\cdot)$, $\#(\cdot)$, $|\cdot|$

If $\exists \phi : A \to B$ & ϕ is bijective, then

card A = card B.

If $\exists \psi : A \to B$ & ψ is injective

card $A \leq$ card B

If $\exists \lambda : A \to B$ & λ is surjective

card $A \geq$ card B

Levels of cardinality

$\aleph_0$ – countabe card $\mathbb{N}$ = card $\mathbb{Z}$ = $\aleph_0$

$\aleph_1$ is next one up ??

Q. is cardinality of continuum $\mathbb{R}$

$\underline{2^{\aleph_0} = \aleph_1}$

$2^{\aleph_\alpha} = \aleph_{\alpha+1}$

GCH