

①

4-17

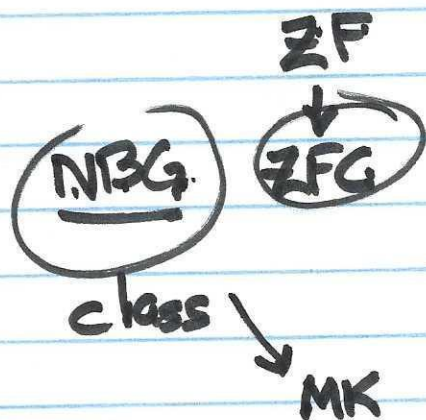
There is no set of all sets.

Pf: FSCC Let $S =$ set of all sets.

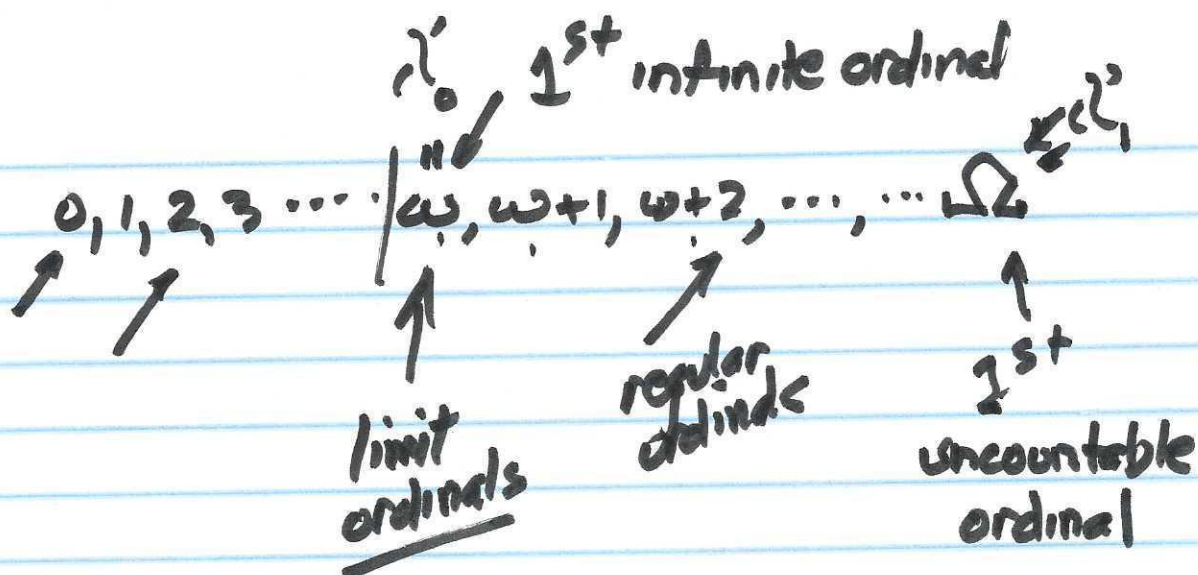
Let $M =$ set of all sets not members of themselves
 $M = \{A \in S : A \notin A\}$

Consider: $M \in M \Rightarrow M \notin M \Rightarrow \text{contradiction}$

Must be the case $M \notin M \Rightarrow \text{contradiction}$



$$\bigcup_{\alpha \in \Lambda} S_{\alpha} \neq \emptyset$$



Class of ordinals $\mathcal{ON}$

Burali-Forti Paradox

$$1 + \omega = \omega$$

$$\omega + 1 \neq \omega$$

$$^*\mathbb{R} > \mathbb{R}$$

Sequences illustrating limits:

- $(1, \frac{1}{2}, \frac{1}{3}, \dots \rightarrow 0)$
- $(1, \frac{1}{2}, \frac{1}{3}, \dots \rightarrow 0)$
- $(0, 0, 0, \dots)$

③

Th^m ~~card A~~ $\text{card}(2^A) \geq \text{card } A$

Pf: Certainly $\text{card } A \leq \text{card } 2^A$

FSCC Suppose $\text{card } A = \text{card } 2^A$

$\exists \phi : A \rightarrow 2^A$ which is bijective

Let $a \in A$, if $a \in \phi(a) \Rightarrow a$ is "good"

$a \notin \phi(a) \Rightarrow a$ is "bad".

Consider $B = \{\text{bad elements}\}$

$\phi(b) = B$

Is b good or bad?

b not good / b is not bad $\Rightarrow \neq$

Cardinality & Ordinality

$\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\} \rightarrow$

0 1 2 3 4

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$\text{card}(\cdot), \#(\cdot), |\cdot|$

If $\exists \phi: A \rightarrow B$; ϕ is bijective, then
 $\text{card } A = \text{card } B$.

If $\exists \psi: A \rightarrow B$; ψ is injective
 $\text{card } A \leq \text{card } B$

If $\exists \lambda: A \rightarrow B$; λ is surjective
 $\text{card } A \geq \text{card } B$

Levels of cardinality

$\aleph_0$ - countable $\text{card } \mathbb{N} = \text{card } \mathbb{Z} = \aleph_0$

$\aleph_1$ is next one up ??

$\mathfrak{c}$ is cardinality of continuum $\mathbb{R}$

$$\underline{2^{\aleph_0} = \aleph_1}$$

$$2^{\aleph_\alpha} = \aleph_{\alpha+1}$$

GCH