T_k^{ij}

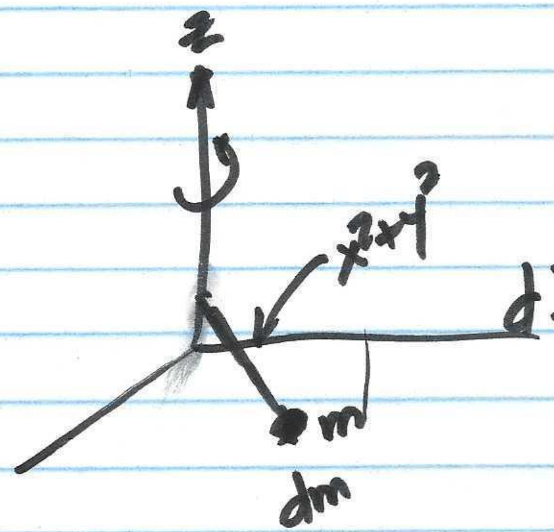
$x \to x_1$
$y \to x_2$
$z \to x_3$

$$\bar{T}_{k'}^{i'} = \sum \sum \frac{\partial}{\partial x} \frac{\partial}{\delta x} T_k^i$$

$$\bar{T}_{k'}^{i'} = \sum_i \sum_k \frac{\partial x_k}{\partial \bar{x}_{k'}} \frac{\partial \bar{x}_{i'}}{\partial x_i} T_k^i$$

cos

$$\bar{V}_i = \sum_k \frac{\partial x_k}{\partial x_i} V_k$$



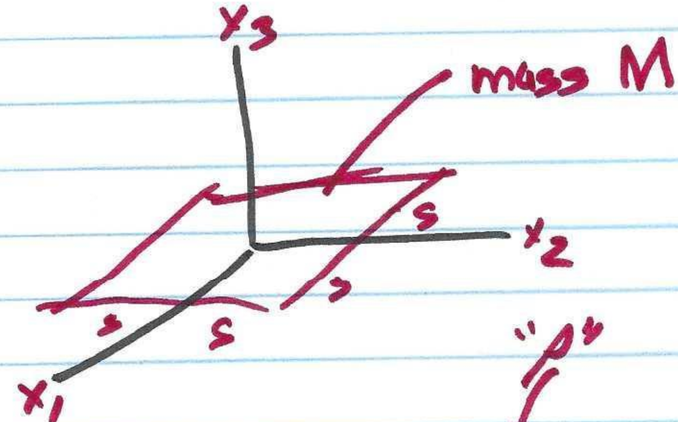
$$dI_z = (x^2+y^2)\,dm$$

$$\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}$$

Principal MOI

$$I_{ij} = \iiint_V \left(r^2 \delta_{ij} - x_i x_j\right) \rho(r)\, \underbrace{dx^3}_{dx_1 dx_2 dx_3}$$

Thin Square Sheet - Scalar Case



$$I = \int_{-s}^{s}\int_{-s}^{s} (x_1^2 + x_2^2)\, \underbrace{\frac{M}{4s^2}}_{\text{"}\rho\text{"}}\, dx_1 dx_2$$

Scalar moment of inertia about x_3 axis is

$$dI = (x_1^2 + x_2^2)\,dm$$

$$I = \frac{2}{3} s^2 M \Rightarrow \hat{L} = \frac{2}{3} s^2 M \hat{\omega} \, .$$

$$I_{11} = \frac{M}{4s^2} \iint (r^2 - x_1^2)\, d^2x = \frac{1}{3} s^2 M$$

$$I_{22} = \frac{1}{3} s^2 M$$

$$I_{33} = 0$$

Off-diagonal (Products of Inertia)

$$I_{13} = I_{31} = 0 = I_{12} = I_{21}$$

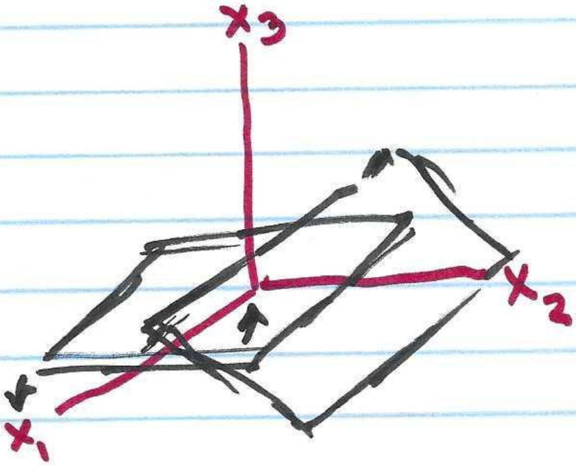
$$I_{ij} = \frac{s^2 M}{3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

What is $\mathbb{L}$ if $\begin{bmatrix}0\\0\\\omega\end{bmatrix}$

$$\mathbb{L} = [\mathbb{I}]\begin{bmatrix}0\\0\\\omega\end{bmatrix}$$

$$\mathbb{L} = \frac{s^2M}{3}\begin{bmatrix}1&0&0\\0&1&0\\0&0&2\end{bmatrix}\begin{bmatrix}0\\0\\\omega\end{bmatrix} =$$

$$\mathbb{L} = \frac{1}{3}s^2M\begin{bmatrix}0\\0\\2\omega\end{bmatrix} = \frac{2}{3}s^2M\omega\begin{bmatrix}0\\0\\1\end{bmatrix}.$$



④

$$\bar{I}_{ij} = \sum_k \sum_l \frac{\partial x^k}{\partial \bar{x}^i} \frac{\partial x^l}{\partial \bar{x}^j} I_{kl}$$

X-form in plane - $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{6} & \sin\frac{\pi}{6} \\ 0 & -\sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix} \quad \theta = \frac{\pi}{6}$$

$\frac{S^2 M}{3}$

$$\bar{I}_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

5

$$\overline{I}_{ij} = \frac{Ms^2}{12}\begin{bmatrix} 2 & 0 & 0 \\ 0 & \sqrt{3} & -1 \\ 0 & 1 & \sqrt{3} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & \sqrt{3} & +1 \\ 0 & -1 & \sqrt{3} \end{bmatrix} = 2$$

$$\frac{s^2M}{12}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & -\sqrt{3} \\ 0 & -\sqrt{3} & 7 \end{bmatrix}$$

In general $[I_{ij}] = [T_{ik}][I_{kl}][T_{jl}]^T$

What is $\mathbb{L}$ if sheet is now spun about

x_3 axis. x_3' new axis

$$\omega' = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \quad \text{so}$$

$$\mathbb{L} = \frac{1}{12} s^2M \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & -\sqrt{3} \\ 0 & -\sqrt{3} & 7 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} =$$

$$\frac{1}{12} s^2 M \omega \begin{bmatrix} 0 \\ -\sqrt{3} \\ 7 \end{bmatrix}$$ ← small new component of L about x_2-axis