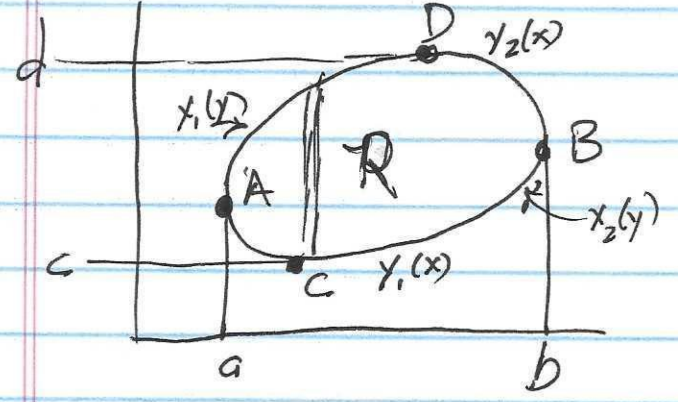
$P, Q, \frac{\partial P}{\partial y}, \frac{\partial P}{\partial x}$ cont. in R



Convex area

$$\iint_R \frac{\partial P}{\partial y}\,dx\,dy = \int_{x=a}^{b} \left[\int_{y=y_1(x)}^{y=y_2(x)} \frac{\partial P}{\partial y}\,dy \right] dx =$$

$$\int_{x=a}^{x=b} \left[P(x,y_2) - P(x,y_1) \right] dx = -\int_a^b P(x,y_1)\,dx - \int_b^a P(x,y_2)\,dx =$$

$$= -\oint_C P\,dx, \quad \text{so} \quad \oint_C P\,dx = -\iint_R \frac{\partial P}{\partial y}\,dx\,dy$$

$$\iint_R \frac{\partial Q}{\partial x}\,dx\,dy = \int_{y=c}^{y=d} \left[\int_{x_1(y)}^{x_2(y)} \frac{\partial Q}{\partial x}\,dx \right] dy =$$

$$\int_{y=c}^{y=d}\left[Q(x_2,y)\,dy - Q(x_1,y)\right]dy$$

$$= \int_c^d Q(x_2 y)\,dy + \int_d^c Q(x_1,y)\,dy = \oint_C Q\,dy,$$

$$\text{so } \oint_C Q\,dy = \iint_R \frac{\partial Q}{\partial x}\,dx\,dy$$

$$\text{Then } \oint_C P\,dx + Q\,dy = \iint_R \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right]dx\,dy$$

③

$Q = x \quad P = -y$

$$\iint_R dx\,dy = \frac{1}{2}\oint_{\partial R}(\overset{P}{-y}\,dx + \overset{Q}{x}\,dy)$$

Show $\mathbf{F} := (2xz^3 + 6y)\,\hat{i} + (6x - 2yz)\,\hat{j} + (3x^2z^2 - y^2)\,\hat{k}$

is conservative

Find work done from $(1,-1,1)$ to $(2,1,-1)$

Any path as $\quad (1,-1,1) \longrightarrow (2,-1,1) \longrightarrow (2,1,1) \longrightarrow (2,1,-1)$

$y=-1$, $z=1$, $dy=0$, $dz=0$ $\qquad$ $x=2$, $z=1$ $\qquad$ $x=2$, $y=1$

$$\int_{x=1}^{2}(2x-6)\,dx + \int_{y=-1}^{1}(12-2y)\,dy + \int_{z=1}^{-1}(12z^2-1)\,dz$$

Ellipse: $A = \frac{1}{2} \oint_C x\,dy - y\,dx =$

$x = a\cos\theta \quad y = b\sin\theta$