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(10) $\nabla(A\cdot B) = ?$

Note: → $\overset{A}{A} \times (\overset{B}{\nabla} \times \overset{C}{B}) = \nabla(A\cdot B) - B(A\cdot\nabla)$

(*) $= \nabla_B(A\cdot B) - (A\cdot\nabla_B)B$

$\nabla = \nabla_A + \nabla_B$

$(\nabla_A + \nabla_B)(A\cdot B)$

(*) $\nabla(A\cdot B) - (A\cdot\nabla)B$

→ $\overset{A}{B} \times (\overset{B}{\nabla} \times \overset{C}{A}) = \nabla_A(A\cdot B) - A(B\cdot\nabla)$

(**) $= \nabla_A(A\cdot B) - (B\cdot\nabla)A$

$\nabla(A\cdot B) = \nabla_B(A\cdot B)$

2

(12) $\nabla \cdot (A \times B) = \begin{vmatrix} \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$= \partial_x \left[A_y B_z - A_z B_y \right] - \partial_y \left[A_x B_z - A_z B_x \right] + 2$$

$$\partial_z \left[A_x B_y - A_y B_x \right]$$

$$= \overset{5}{B_z \partial_x A_y} + A_y \partial_x B_z - \left[\overset{3}{B_y \partial_x A_z} + A_z \partial_x B_y \right] \; 2$$

$$- \left[\overset{6}{B_z \partial_y A_x} + A_x \partial_y B_z - \left(B_x \partial_y A_z + A_z \partial_y B_x \right) \right] 2$$

$$+ B_y \partial_z A_x + \overset{A}{A_x \partial_z B_y} - \left[\underset{2}{B_x \partial_z A_y} + A_y \partial_z B_x \right]$$

3

$$B \cdot \nabla \times A = \begin{vmatrix} B_x & B_y & B_z \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix}$$

$$\overset{1}{B_x(\partial_y A_z} \overset{2}{- \partial_z A_y)} - \overset{3}{B_y(\partial_x A_z} \overset{4}{- \partial_z A_x)} +$$

$$\underset{5}{B_z(\partial_x A_y} \underset{6}{- \partial_y A_x)}$$

(14)

(a)

$$\nabla \times (A \times B) = \overset{A}{\nabla} \times (\overset{B}{\bar{A}} \times \overset{C}{B}) + \overset{A}{\nabla} \times (\overset{B}{A} \times \overset{C}{\bar{B}})$$

$$\downarrow$$

$$\bar{A}(\nabla \cdot B) - B(\nabla \cdot \bar{A})$$

$$+ \; A(\nabla \cdot \bar{B}) - \bar{B}(\nabla \cdot A)$$

$$A(\nabla \cdot B) - (A \cdot \nabla)B + (B \cdot \nabla)A$$

$$- B(\nabla \cdot A)$$

VECTOR IDENTITIES

1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = -\mathbf{A}\cdot(\mathbf{C}\times\mathbf{B}) = -\mathbf{B}\cdot(\mathbf{A}\times\mathbf{C}) = -\mathbf{C}\cdot(\mathbf{B}\times\mathbf{A})$
2) $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{A}\times\mathbf{B}) = (\|\mathbf{A}\|\|\mathbf{B}\|)^2 - (\mathbf{A}\cdot\mathbf{B})^2$
3) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$ (BAC-CAB formula)
4) $(\mathbf{A}\times\mathbf{B})\times\mathbf{C} = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{A}(\mathbf{B}\cdot\mathbf{C})$ (variant)
5) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = \mathbf{0}$ (Jacobi's Identity)
6) $\nabla(\alpha\phi + \beta\psi) = \alpha\nabla\phi + \beta\nabla\psi$ (linearity)
7) $\nabla\cdot(\alpha\mathbf{A} + \beta\mathbf{B}) = \alpha\nabla\cdot\mathbf{A} + \beta\nabla\cdot\mathbf{B}$ (linearity)
8) $\nabla\times(\alpha\mathbf{A} + \beta\mathbf{B}) = \alpha\nabla\times\mathbf{A} + \beta\nabla\times\mathbf{B}$ (linearity)
9) $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$ (product rule for gradients)
10) $\nabla(\mathbf{A}\cdot\mathbf{B}) = (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A} + \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A})$!
11) $\nabla\cdot(\phi\mathbf{A}) = \phi\nabla\cdot\mathbf{A} + \nabla\phi\cdot\mathbf{A}$
12) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$
13) $\nabla\times(\phi\mathbf{A}) = \phi\nabla\times\mathbf{A} + \nabla\phi\times\mathbf{A}$
14) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$
15) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$ (curls have no divergence)
16) $\nabla\times(\nabla\phi) = \mathbf{0}$ (gradients have no curl)
17) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$ definition of vector laplacian