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Vectors in $\mathbb{R}^2, \mathbb{R}^3$

$$\mathbb{A} = A_x \hat{e}_x + A_y \hat{e}_y + A_z \hat{e}_z$$

$$\mathbb{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$$

$$A_x = \mathbb{A} \cdot \hat{e}_x \text{ etc}$$

$$A_x = A\cos\alpha \quad \text{where } A = |\mathbb{A}| = \sqrt{\mathbb{A}\cdot\mathbb{A}}$$

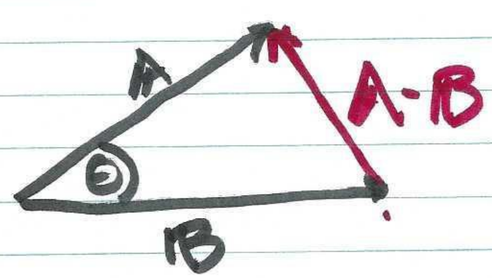
$$A = (A_x^2 + A_y^2 + A_z^2)^{1/2}$$

$$\mathbb{A}\cdot\mathbb{B} = \sum_{ij} A_i B_j \delta_{ij} \qquad \langle A_i \rangle$$

$$A_x B_x + A_y B_y + A_z C_z$$

$$A_1 B_1 + A_2 B_2 + A_3 B_3$$

$$\mathbb{A}\cdot\mathbb{B} = AB\cos\Theta$$ Θ is angle between vectors



Law of Cosines $|\mathbb{A}-\mathbb{B}|^2 = $ -

$$A^2 + B^2 - 2AB\cos\Theta$$

$$|\mathbf{A}-\mathbf{B}|^2 = (\mathbf{A}-\mathbf{B})\cdot(\mathbf{A}-\mathbf{B})$$
$$= \mathbf{A}\cdot\mathbf{A} - 2\mathbf{A}\cdot\mathbf{B} + \mathbf{B}\cdot\mathbf{B}$$
$$= A^2 - 2\mathbf{A}\cdot\mathbf{B} + B^2$$
$$= A^2 + B^2 - 2AB\cos\theta$$

$\div(-2)$ $\quad \mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

$$\mathbf{A}\cdot\mathbf{B} = 0 \Rightarrow \mathbf{A} \perp \mathbf{B}$$

$$\mathbf{A} = \begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix}$$

$$\mathbf{A}\cdot\mathbf{B}^T = \begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$$

$\ast$ $\quad \mathbf{A}\cdot\mathbf{B} = \sum_{ij} A_i B_j \delta_{ij}$

$\ast$ $\quad \mathbf{A}\times\mathbf{B} = \mathbf{C} \qquad C_i = \sum_{jk} \varepsilon_{ijk} A_j B_k$

Levi-Civita

$$\varepsilon_{ijk} = \begin{cases} +1 & \text{if } ijk \sim 123 \\ -1 & \text{if } ijk \sim 132 \\ 0 & \text{if 2 indices same} \end{cases}$$

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$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\hat{e}_x\left(A_yB_z - A_zB_y\right) + \cdots$$

$$|\mathbf{A}\times\mathbf{B}| = AB\sin\theta$$

$$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A}$$

-1

$$\mathbf{A}\cdot\mathbf{C}\times\mathbf{B} = \quad\cdot\quad\cdot$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = ?$$

$$(\mathbf{B}\times\mathbf{C})_k = \varepsilon_{\underline{kpq}}\,\hat{e}_k\,\underline{B_pC_q}$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \varepsilon_{ijk}\,\hat{e}_i A_j (\mathbf{B}\times\mathbf{C})_k$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \varepsilon_{ijk}\,\varepsilon_{kpq}\,\hat{e}_i A_j B_p C_q$$

$$\underline{\varepsilon_{ijk}\,\varepsilon_{kpq} = \delta_{ip}\,\delta_{jq} - \delta_{iq}\,\delta_{jp}}$$

$$\delta_{ip}\,\delta_{jq}\,\hat{e}_i A_j B_p C_q - \delta_{iq}\,\delta_{jp}\,\hat{e}_i A_j B_p C_q$$

$$\hat{e}_i A_j B_i C_j - \hat{e}_i A_j B_j C_i$$

$$\underline{\hat{e}_i B_i}\,(A_j C_j) - \underline{\hat{e}_i C_i}\,(A_j B_j)$$

$$\mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$$

<u>BAC-CAB</u>

$$\nabla = \hat{e}_i \frac{\partial}{\partial x_i} \qquad \mathbf{F} = -\nabla\phi$$

$$\nabla\phi(x,y,z) \qquad d\mathbf{r} = \hat{e}_i\,dx_i$$

$$d\phi = \nabla\phi\cdot d\mathbf{r} = \frac{\partial\phi}{\partial x_1}dx_1 + \frac{\partial\phi}{\partial x_2}dx_2 + \frac{\partial\phi}{\partial x_3}dx_3$$

gradient of ϕ $\nabla\phi$
divergence of V $\nabla\cdot V$
curl of V $\nabla\times V$

1) $\nabla\cdot\nabla\phi = \nabla^2\phi$

Laplacian

2) $\nabla\times\nabla\phi = 0$

3) $\nabla(\nabla\cdot V)$

$\nabla^2\phi = \underline{\Delta\phi}$
old

4) $\nabla\cdot(\nabla\times V) = 0$

$\nabla^2 V$

→ 5) $\nabla\times(\nabla\times V) = \nabla(\nabla\cdot V) - \boxed{\nabla\cdot\nabla V}$

special vector laplacian

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Maxwell

E is elec B is mag

Lorentz equation

$F = q(E + v\times B)$

1) $\nabla\cdot E = \rho/\varepsilon_0$

2) $\nabla\times E = -\frac{\partial B}{\partial t}$

3) $\nabla\cdot B = 0$

4) $c^2(\nabla\times B) = \frac{J}{\varepsilon_0} + \frac{\partial E}{\partial t}$