$$f(t) = r^n \text{ if } n \le t < n+1$$

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\,dt = $$

$$\int_0^1 (1)e^{-st}\,dt + \int_1^2 (r)e^{-st}\,dt + \int_2^3 (r^2)e^{-st}\,dt + \cdots$$

$$\left[\frac{e^{-st}}{-s}\right]_0^1 + r\left[\frac{e^{-st}}{-s}\right]_1^2 + r^2\left[\frac{e^{-st}}{-s}\right]_2^3 + \cdots$$

$$\downarrow$$

$$\frac{e^{-s}-1}{-s} + r\left[\frac{e^{-2s}-e^{-s}}{-s}\right] + r^2\left[\frac{e^{-3s}-e^{-2s}}{-s}\right] + \cdots$$

$$\frac{1-e^{-s}}{s} + r\left(\frac{e^{-s}-e^{-2s}}{s}\right) + r^2\left(\frac{e^{-2s}-e^{-3s}}{s}\right) \cdots$$

$$\downarrow$$

[illegible]

$$\downarrow$$

$$re^{-s}\left(\frac{1-e^{-s}}{s}\right) + r^2e^{-2s}\left(\frac{1-e^{-s}}{s}\right) + \cdots$$

(2)

$$= \frac{1-e^{-s}}{s}\left[1 + re^{-s} + r^2e^{-2s} + r^3e^{-s} + \cdots\right]$$

$$\downarrow$$

$$\underline{\frac{1-e^{-s}}{s}}\left(\frac{1}{1-re^{-s}}\right) = \frac{1-e^{-s}}{s(1-re^{-s})}$$

$f(t+1)$ if $\underline{f(t) = a_n}$ for $n \le t < n+1$

$$\mathcal{L}(f(t+1)) = \int_0^\infty e^{-st} f(t+1)\,dt \overset{u=t+1}{=}$$

$$\int_{u=1}^{\infty} e^{-s(u-1)} f(u)\,du = e^{s}\int_1^\infty e^{-su} f(u)\,du =$$

$$\underbrace{e^{s}\int_0^\infty e^{-su} f(u)\,du}_{F(s) = \mathcal{L}(f(t))} - e^{s}\int_0^1 e^{-su} f(u)\,du$$

$$= e^{s}F(s) \;\ldots\ldots\; - e^{s}a_0 \underbrace{\int_0^1 e^{-su}\,du}_{\left.\frac{e^{-su}}{-s}\right|_0^1}$$

3

$$\mathcal{L}(f(t+1)) = e^{s}F(s) - e^{s}a_0\left(\frac{1-e^{-s}}{s}\right)$$

$$\mathcal{L}(f(t+2)) = \int_0^\infty e^{-st} f(t+2)\,dt \;\xrightarrow{u=t+2}\; = 2$$

$$\int_{2\,\cancel{0}}^{\infty} e^{-s(u-2)} f(u)\,du = e^{2s}\int_{2\,\cancel{0}}^{\infty} e^{-su} f(u)\,du$$

$$e^{2s}\int_2^\infty e^{-su} f(u)\,du = e^{2s}\int_0^\infty e^{-su} f(u)\,du - e^{2s}\int_0^1 e^{-su} \overset{a_0}{f(u)}\,du -$$

$$e^{2s}\int_1^2 e^{-su} \overset{a_1}{f(u)}\,du \quad = 2$$

$$e^{2s}F(s) - a_0 e^{2s}\left[\frac{e^{-su}}{-s}\right]_0^1 - a_1 e^{2s}\left[\frac{e^{-su}}{-s}\right]_1^2$$

4

$$= e^{2s}F(s) - a_0 e^{2s}\left(\frac{1-e^{-s}}{s}\right) - a_1 e^{2s}\left(\frac{e^{-s}-e^{-2s}}{s}\right)$$

$$= e^{2s}F(s) - \left(\frac{1-e^{-s}}{s}\right)\left[a_0 e^{2s} + a_1 e^{s}\right]$$

$$= e^{2s}F(s) - \frac{e^{s}(1-e^{-s})}{s}\left[a_0 e^{s} + a_1\right]$$

$$a_{n+2} - 5a_{n+1} + 6a_n = 0 \quad \begin{array}{|l} a_0 = 0 \\ a_1 = 1 \end{array}$$

Let $f(t) = a_n$

$$f(t+2) - 5f(t+1) + 6f(t) = 0$$

$$\left[e^{2s}F(s) - \frac{e^{s}(1-e^{-s})(1)}{s}\right] - 5\left[e^{s}F(s)\right] + 6F(s) = 0$$

5

$$F(s)\left[e^{2s} - 5e^{s} + 6\right] = \frac{e^{s}(1-e^{-s})}{s}$$

$$F(s) = \frac{e^{s}(1-e^{-s})}{s(e^{s}-3)(e^{s}-2)} = ?$$

$$\frac{e^{s}(1-e^{-s})}{s} \cdot \left[\frac{1}{e^{s}-3} - \frac{1}{e^{s}-2}\right]$$

$\div e^{s}$

$$\frac{e^{s}(1-e^{-s})}{s} \cdot \frac{1}{e^{s}-3} - \frac{e^{s}(1-e^{-s})}{s} \cdot \frac{1}{e^{s}-2}$$

$$\underbrace{\left(\frac{1-e^{-s}}{s}\right)\left(\frac{1}{1-3e^{-s}}\right)} - \left(\frac{1-e^{-s}}{s}\right)\left(\frac{1}{1-2e^{-s}}\right)$$

$\mathcal{L}^{-1}$

$$a_n = 3^n - 2^n$$

$$a_{10} = 3^{10} - 1024$$

$$59{,}049 - 1024 = 58{,}025$$