2/25

$$ty''(t) + y'(t) + 4ty(t) = 0 \quad \left|\begin{array}{l} y(0)=0 \\ y'(0)=0 \end{array}\right.$$

$\downarrow$

$$\mathcal{L}(ty''(t)) \cdot (-1)\frac{d}{ds}\left(s^2Y - sy(0) - y'(0)\right)$$

$$\mathcal{L}(y'(t)) = sY - y(0)$$

$$A \cdot (-1)\frac{d}{ds}(Y)$$

$$-\frac{d}{ds}s^2Y + sY - 4\frac{d}{ds}Y = 0$$

$$2sY + s^2\frac{dY}{ds} - sY + 4\frac{dY}{ds} = 0$$

$$(s^2+4)\frac{dY}{ds} + sY = 0$$

$$(s^2+4)\frac{dY}{ds} = -sY$$

$$u = s^2+4$$
$$du = 2s\,ds$$

$$\frac{dY}{Y} = -\frac{s}{s^2+4}\,ds$$

$$= -\frac{1}{2}\left(\frac{2s\,ds}{s^2+4}\right)$$

②

$$\ln Y = -\frac{1}{2}\ln(s^2+4) + C_0$$

$$Y = \frac{1}{\sqrt{s^2+4}}\, e^{C_0} = \frac{C_1}{\sqrt{s^2+4}} = Y(s)$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{C_1}{\sqrt{s^2+4}}\right) = C_1 J_0(2t)$$

$$y(0) = C_1 J_0(0) = 0$$

$$z^2 w''(z) + z w'(z) + (z^2 - n^2) w = 0$$

$$J_n(z) \quad .$$

1) $x'(t) = 2x - 3y$ $\qquad x(0) = 8$

2) $y'(t) = y - 2x$ $\qquad y(0) = 3$

1') $sX - 8 = 2X - 3Y$

2') $sY - 3 = Y - 2X$ $\qquad 2X + (s-1)Y = 3$

1'') $(s-2)X + 3Y = 8$

2'') $2X + (s-1)Y = 3$

$$X = \frac{\begin{vmatrix} 8 & 3 \\ 3 & (s-1) \end{vmatrix}}{\begin{vmatrix} (s-2) & 3 \\ 2 & (s-1) \end{vmatrix}} = \frac{8(s-1)-9}{(s-2)(s-1)-6} = \frac{8s-17}{(s-4)(s+1)}$$

$$Y = \frac{\begin{vmatrix} (s-2) & 8 \\ 2 & 3 \end{vmatrix}}{\begin{vmatrix} (s-2) & 3 \\ 2 & (s-1) \end{vmatrix}} = \frac{3(s-2)-16}{(s-2)(s-1)-6} = \frac{3s-22}{(s-4)(s+1)}$$

$$X = \frac{3}{s-4} + \frac{5}{s+1}$$

$$\frac{3s+3+5s-20}{}$$

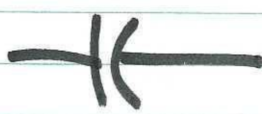
$$Y = \frac{5}{s+1} - \frac{2}{s-4}$$

Conclude $\left.\begin{aligned} x(t) &= 3e^{4t} + 5e^{-t} \\ y(t) &= 5e^{-t} - 2e^{4t} \end{aligned}\right\}$

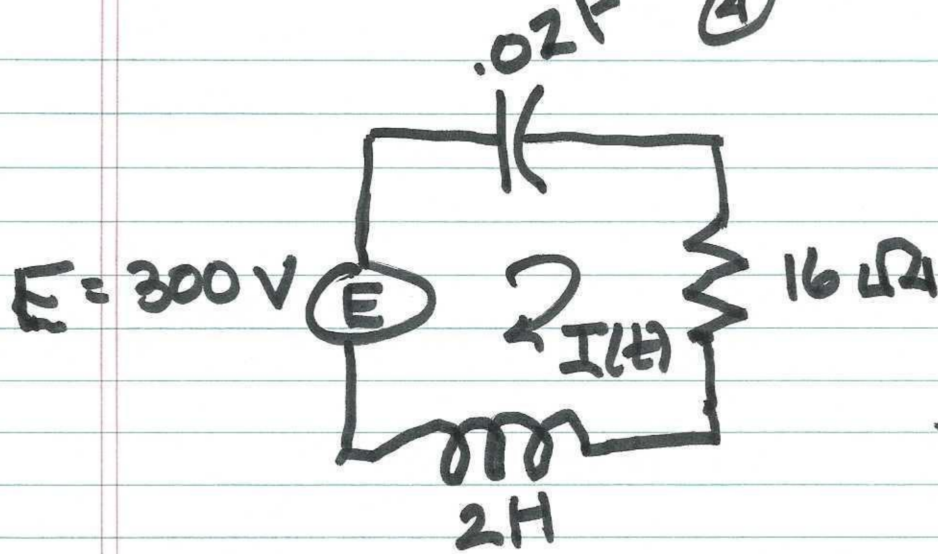
Electrical Problems:

 resistance Ω

 inductance

 capacitance

④



$\rightarrow I(0) = 0$

$Q(0) = 0$ is charge

$$V_R = IR \qquad V_C = \frac{Q}{C} \qquad V_L = +L\frac{dI}{dt}$$

$$I(t) = Q'(t)$$

$$* \quad L\frac{dI}{dt} + RI + \frac{Q}{C} = E$$

$$\boxed{L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E}$$

$$2\frac{d^2Q}{dt^2} + 16\frac{dQ}{dt} + \frac{Q}{.02} = 300$$

$$* \quad \frac{d^2Q}{dt^2} + 8\frac{dQ}{dt} + 25Q = 150$$

$$\rightarrow \frac{d^2q}{dt^2} + 8\frac{dq}{dt} + 25q = 150$$

5

$$\left(s^2Q - s\cancelto{0}{q(0)} - \cancelto{0}{q'(0)}\right) + 8\left(sQ - \cancelto{0}{q(0)}\right) + 2$$

$$25Q = \frac{150}{s}$$

$$s^2Q + 8sQ + 25Q = \frac{150}{s}$$

$$Q\left(s^2 + 8s + 25\right) = \frac{150}{s}$$

$$Q = 150 \cdot \frac{1}{s(s^2+8s+25)}$$

$$= 150 \cdot \frac{3}{3s\left[(s+4)^2+3^2\right]}$$

$$\underline{150} \quad \frac{-s-8}{25(s^2+8s+25)} + \frac{1}{25s}$$

$$\cancel{25s}$$

$$\frac{6}{s} - \frac{6(s+8)}{s^2+8s+25}$$

$$6 -$$

⑥

$$\frac{s+8}{s^2+8s+25} = \frac{s}{s^2+8s+25} + \frac{8}{s^2+8s+25}$$

$$\downarrow \qquad \downarrow$$

$$\frac{s}{(s+4)^2+3^2} \qquad \frac{3 \cdot 8/3}{(s+4)^2+3^2}$$

$$\mathcal{L}^{-1} \downarrow$$

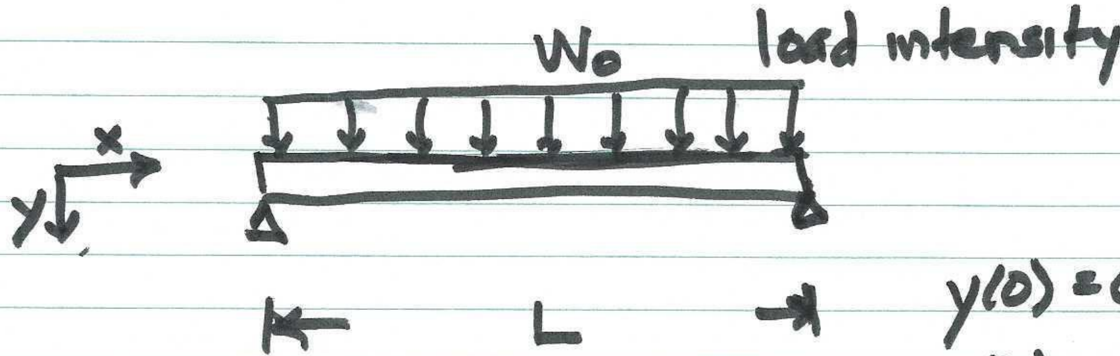
$$q(t) = 6 - 6\left[e^{-4t}\cos 3t + \frac{8}{3}e^{-4t}\sin 3t\right]$$

$$q(t) = 6 - 6e^{-4t}\left[\cos 3t + \frac{8}{3}\sin 3t\right]$$

$$I(t) = q'(t) = \left[24e^{-4t}\left[\cos + \sin\right] -\right.$$

$$\underline{I(t) = 6e^{-4t}\left[-3\sin 3t + 8\cos 3t\right]}$$

7



$y(0) = 0$
$y(L) = 0$
$y''(0) = 0$
$y''(L) = 0$
$y'(0), y'(L) \equiv$

EI

$$\frac{d^4 y}{dx^4} = \frac{W_0}{EI}$$