U4SZM

9/17

1

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 2 & 1 & 4 \end{bmatrix} \qquad \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 \\ \beta_5 & \cdots & & & \end{bmatrix}$$

$$|S_5| = 5! = 120$$

$f \circ g(x)$

$f(g(x))$

$$\alpha^{-1} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 2 & 5 & 1 \end{bmatrix}$$

$$\underline{\alpha^{-1}\alpha} = \alpha\alpha^{-1}$$

$$\begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 \\ \alpha & 5 & 3 & 2 & 1 & 4 \\ \alpha^{-1} & \boxed{1} & \boxed{2} & \boxed{3} & \boxed{4} & \boxed{5} \end{array}$$

same

$(1), (3)$

Reduce to cycle notation:

$$\alpha = (1\ 5\ 4)(2\ 3)$$

3x 2x

$$\alpha^6 = (1)$$

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{bmatrix} \qquad |S_6| = \underline{720}$$

Find α^{-1} $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 6 & 3 & 5 & 4 \end{bmatrix}$

$|\alpha|$

$$\alpha = (1\ 2)(3\ 4\ 6)(5)$$

$$\alpha^{-1} = (1\ 2)(4\ 3\ 6)(5)$$

4 3 6
3 6 4
6 4 3

$$|\alpha| = 6$$

$$|g| = n \quad \overbrace{ggg\cdots g}^{n} = e$$

$$\rightarrow ggg\cdots \underbrace{\cancel{g g^{-1}}}_{e} = g^{-1} \longrightarrow$$

$$e = (g^{-1})^n$$

3

$\alpha \in S_8$

$\alpha = (13)(27)(456)(8)(1237)(648)(5)$

$$
\begin{array}{l|cccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
(5) & & & & & \downarrow & & & \\
(648) & 1 & 2 & 3 & 8 & 5 & 4 & 7 & 6 \\
(1237) & 2 & 3 & 7 & 8 & 5 & 4 & 1 & 6 \\
(8) & & & & & & & & \\
 & 2 & 3 & 7 & 8 & 6 & 5 & 1 & 4 \\
 & 7 & 3 & 2 & 8 & 6 & 5 & 1 & 4 \\
 & 7 & 1 & 2 & 8 & 6 & 5 & 3 & 4
\end{array}
$$

$$
\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 1 & 2 & 8 & 6 & 5 & 3 & 4 \end{bmatrix}
$$

$\alpha = (1\,7\,3\,2)(4\,8)(5\,6)$

$|\alpha| = \operatorname{lcm}(3,2,4) = \boxed{4}$

transposition model # 1

$$(a_1\, a_2\, a_3 \cdots a_k) = (\quad)(\quad)(\quad)$$

$$\to (a_1 a_k)(a_1 a_{k-1}) \cdots (a_1 a_3)(a_1 a_2)$$

$$\alpha : (1\ 3\ 2\ 5\ 4)$$

$$= \underline{(1\,4)(1\,5)(1\,2)(1\,3)}$$

$$\alpha : \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 2 & 1 & 4 \end{bmatrix} \checkmark$$

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 4 & 5 \\ 3 & 1 & 2 & 4 & 5 \\ 3 & 5 & 2 & 4 & 1 \\ 3 & 5 & 2 & 1 & 4 \end{matrix}$$

Model 2:

$$(a_1\, a_2\, a_3 \cdots a_{k-1}\, a_k)$$

$$(a_k\, a_{k-1})(a_k\, a_{k-2}) \cdots (a_k\, a_1)$$

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 2 & 1 & 4 \end{bmatrix} = (1\ 3\ 2\ 5\ 4)$$

$$(4\ 5)(4\ 2)(4\ 3)(4\ 1)$$

1	2	3	4	5
4	2	3	1	5
3	2	4	1	5
3	4	2	1	5
3	5	2	1	4

ε_{ijk} Levi-Civita symbol

$$= \begin{cases} 1 & \text{if } ijk \text{ is even perm of } 123 \\ 0 & \text{if any 2 indices are same} \\ -1 & \text{if } ijk \text{ is odd perm of } 123 \end{cases}$$

$$\rightarrow \quad A \times B = \boxed{\varepsilon_{ijk}\, \hat{e}_i A_j B_k}$$

$$A \cdot B = \delta_{ij} A_i B_j$$

Kronecker

Even	Odd
(1 2 3)	(1 3 2)
(1)	(2 1 3)
(3 1 2)	(3 2 1)
(2 3 1)	

The group of permutations that are <u>even</u> in S_n is called A_n (alternating group)

$$|A_n| = \frac{n!}{2}$$

$\underline{R_{\text{cube}}}$

$\alpha = (1\,2\,3\,5)(4\,1\,3)$

	1	2	3	4	5
(413)	3	2	4	1	5
(1235)	5	3	4	2	1

$= (1\,5)(2\,3\,4)$

What is order of:

$$(1\ 2\ 3\ 5)(2\ 4\ 5\ 6\ 7)$$

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 3 & 5 & 6 & 7 & 2 \\ 2 & 4 & 5 & 1 & 6 & 7 & 3 \end{array}$$

$(\_\_\_)$

$(-)$

$$(1\ 2\ 4)(3\ 5\ 6\ 7)(8)$$

$$(1\ 2)(1\ 2) = e$$

$$(8\ 1)(1\ 8)$$