GW8E9

8.20
(1)

Solve: $x^3 + 6x - 20 = 0$

Let $x = u + v$

$$(u+v)^3 + 6(u+v) - 20 = 0$$

$$= u^3 + 3u^2v + 3uv^2 + v^3 + 6(u+v) - 20 = 0$$

$$= u^3 + \underline{3uv(u+v)} + \underline{6(u+v)} + v^3 - 20 = 0$$

Set $3uv + 6 = 0 \Rightarrow uv = -2$

Now original equation is:

$$u^3 + v^3 - 20 = 0$$

But we know $v = \frac{-2}{u}$

$$u^3 + \left(\frac{-8}{u^3}\right) = 20$$

$$(u^3)^2 - 20u^3 - 8 = 0$$

$$u^3 = \frac{20 \pm \sqrt{400 - (4)(1)(-8)}}{2}$$

$$u^3 = \frac{20 \pm \sqrt{432}}{2} = 10 \pm \sqrt{108}$$

+ sq rt for u

- sq rt for v

$$x = \left(10 + \sqrt{108}\right) + \left(10 - \sqrt{108}\right)$$

$$\downarrow$$

$$2.7321\ldots \qquad -.7321\ldots = 2$$

Now solve $x^3 + px + q = 0$

How do you always get this "depressed" form

$$ax^3 + bx^2 + cx + d = 0$$

First, divide by a

$$x^3 + b'x^2 + c'x + d' = 0$$

$x = y - \frac{b}{3}$ von Tschirnhaus substitution

Try this ...

Could get $x^5 + x + 1 = 0$

$$u = \sqrt[5]{x^5 + x + 1}$$

$$\sqrt{x^5 + x + 1}$$

$x^5 - 1 = 0$ (1)

Gen'l quintic $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$

Sets
↓
Groups
↓
Rings → Module
↓
Field
↓
Vector space
↓
Algebra

Axioms for groups $\langle G, * \rangle$ $g_i \in G$

i) Associativity $g_1 * (g_2 * g_3) = (g_1 * g_2) * g_3$

Ex of non-associativity

$$A \times (B \times C) \neq (A \times B) \times C$$

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2) Identity $\exists e \in G \quad e * g = g$

$g * e = g$

3) Inverse $g^{-1} * g = e \mid g * g^{-1} = e$

$\langle G, * \rangle$ is a set (of elements) with properties (1) to (3)

Proof of uniqueness of identity:

Suppose there are two elements e, f that act as identities.

$$\left.\begin{array}{l} e * f = e \\ e * f = f \end{array}\right\} =$$

Extra axiom

4) $g_1 * g_2 = g_2 * g_1$, commutativity

Nils Abel (abelian)

Set : G

Operation (*) binary

$* : G \times G \to G$

(× ↑ cartesian)

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Add Associativity ——— semi-group

Identity ——— monoid

Inverses ——— group ←

Optional Commutativity ——— abelian group

(7) ← Zahlen $\mathbb{N}$

Given $m, n \in \mathbb{Z}$

$m \equiv n \pmod k$ iff

$m - n = rk$ ← modulus

modular arithmetic ≡ "clock" arithmetic

Ex: What are last two digits of 7^{138}

$7^2 = 49$

$49^2 = 2401$

$7^4 = 2401$

$7^8 = \ldots\ldots 01$

But $138 = 4 \cdot 34 + 2$

$7^{4 \cdot 34 + 2} = 7^{4 \cdot 34} \cdot 49$

$\ldots\ldots 01 \cdot 49 = \ldots\ldots 49$

$m \equiv n \pmod k$ is it true $m + a \equiv n + a \pmod k$