HN3AC 8/18

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Given sets A & B, the cartesian product of A & B is:

$$A \times B = \{(a,b) : a \in A, b \in B\}$$

$$(a,b) = \underline{\{\{a\}, \{a,b\}\}}$$

Relation (between) A & B is a subset of $A \times B$.

Relation (on) A is a subset of $A \times A$

* Equivalence relation (on A) is a subset of $A \times A$ w/ 3 properties.

1) reflexive $(a,a) \in R \ \forall a \in A$

$a \sim a$

2) symmetric if $(a,b) \in R \Rightarrow (b,a) \in R$

3) transitivity if (a,b) & $(b,c) \in R \Rightarrow$

$(a,c) \in R$

* Order relation on A $a \leq b$ ($\leq$)

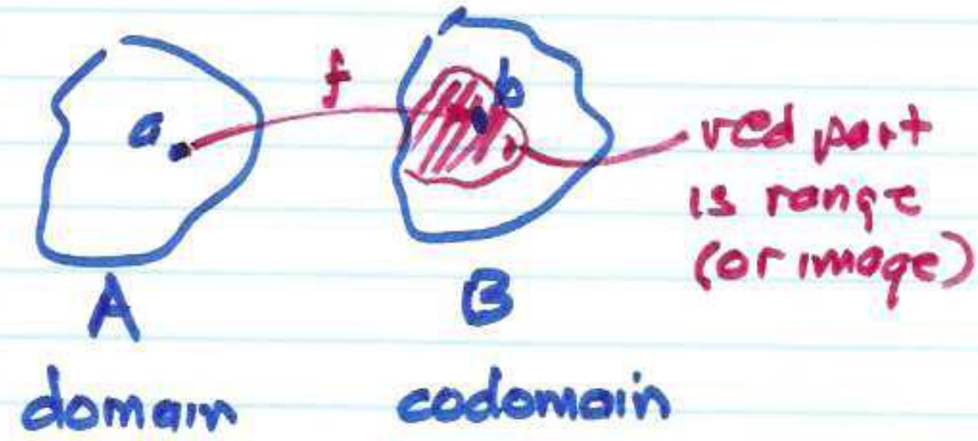
1) $a \leq a$ reflexive

2) $a \leq b$ & $b \leq a \Rightarrow a = b$ antisymmetry

3) $a \leq b$ & $b \leq c \Rightarrow a \leq c$ transitivity

A function from A to B is a subset of the relation $R = \{(a,b) : a \in A, b \in B\}$ such that if (a,b) and $(a,c) \in R$ $b = c$.

Types of functions:



$f(a) = b$

If... $(a,b) \in f$ and $(c,b) \in f$ implies $a = c$, f is injective (1:1) also called invertible f^{-1}

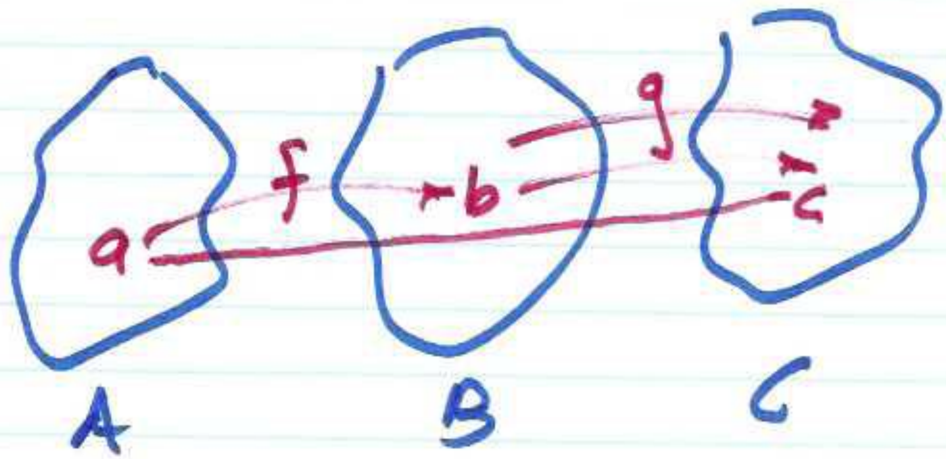
If... $\exists a \in A$ for every $b \in B$ such that $(a,b) \in f$, we say f is surjective

If f is both surjective & injective, we say f is bijective

④

Given f, g functions (injective), is $f \circ g$ or $g \circ f$ injective?

$f(g(x)) \equiv [f \circ g](x)$

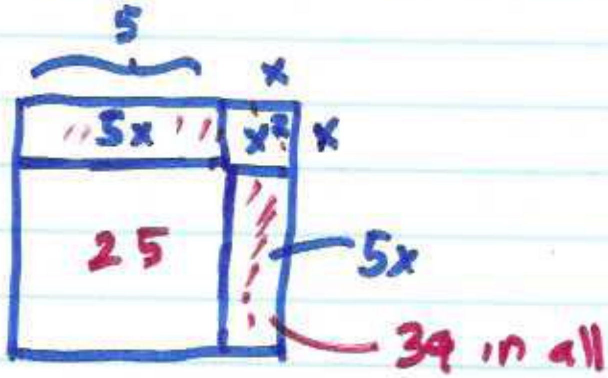


$f \circ g$: f is postcomposed with g

g is precomposed with f

Solve $x^2 + 10x = 39$

Solution (830 AD style)



Big square area $= 39 + 25 = 64$

So sides are both 8

So $x + 5 = 8 \Rightarrow \underline{x = 3}$

Sridhara's Method:

Consider $(2ax+b)^2 = 4a^2x^2 + 4abx + b^2$

$4a(\underbrace{ax^2+bx+c}_{=\,0}) = 4a^2x^2 + 4abx + 4ac$

6

$$(2ax+b)^2 - b^2 + 4ac = 4a(\underbrace{ax^2+bx+c}_{0}) = 0$$

$$(2ax+b)^2 = b^2 - 4ac$$

$$2ax + b = \pm\sqrt{b^2-4ac}$$

$$2ax = -b \pm \sqrt{b^2-4ac}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Solve $x^3 + 6x - 20 = 0$