

6 LGDF

10/13

①

$$3) (abcde \dots n) = (a n) (a n-1) (a n-2) \dots (a b)$$

$$(ab \dots n) = (ab)(ac) \dots (an)$$

$$(1 \ 2 \ 3 \ 4 \ 5) (6 \ 7 \ 8 \ 9)$$

↑
#1

#2

$$4) \underset{2}{(a \ b)} \underset{3}{(c \ d \ e)} \quad \underline{\underline{(c \ d \ e) (c \ d \ e)}}$$

$\binom{5}{2}$ ways of writing $(a \ b)$

$$\# \text{ of } \pi \mid |\pi|=6 \leq 2 \cdot \binom{5}{2} = \frac{5 \cdot 4}{2} = 10$$

$$2 \cdot 10 = \boxed{20}$$

②

Direct Products of Groups (New groups from old)

$$G_1, G_2 \quad G_1 \oplus G_2 =$$

$$\{(g_1, g_2) : g_1 \in G_1, g_2 \in G_2\}$$

x-y plane is $\mathbb{R} \oplus \mathbb{R}$

x-y-z space is $\mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R}$

What is group operation on $G_1 \oplus G_2$ -
component-wise

$$\mathbb{Z}_5 \oplus S_3$$

$$(2, F_1)$$

$$(3, \overset{*}{F}_1)$$

$$\underline{(0, R_0)}$$

$$\bigoplus_{i=1}^n G_i = G_1 \oplus G_2 \oplus \cdots \oplus G_n$$

③

1) We can form the cartesian product of

groups $\prod_{i=1}^n G_i = G_1 \times G_2 \times \dots \times G_n$

2) in the event that $n < \infty$, the direct product and direct sum are the same

3) If $\prod_{i=1}^{\infty} G_i$ then if tuples are allowed

non-identity elements in all positions,

we call the structure a direct product

4) If tuples are allowed non-identity elements in only finitely many positions it is called a direct sum.

⊗ $(\mathbb{Z}_3, \dots, \mathbb{Z}_3) \ni (2, 2, 2, \dots, 2, \dots)$

⊕ " $\ni (2, 2, 0, 0, 0, \dots)$

④

External means groups are arbitrary

Internal means a single group can be represented as a direct product of its own subgroups:

$$H, J \leq G \text{ then if } G \cong H \times J$$

then this is an internal direct product

Thm If $G = G_1 \oplus G_2$ then

$$|G| = |G_1| \cdot |G_2|$$

If (a, b) is in G , then $a \in G_1, b \in G_2$

So there are $|G_1|$ choices for a and

$|G_2|$ choices for b , hence ~~are~~ $|G_1| \cdot |G_2|$

choices for $(a, b) \in G$. $\square$

(5)

Groups of order 4 : $\mathbb{Z}_4$ and $\mathbb{Z}_2 \oplus \mathbb{Z}_2$

$\mathbb{Z}_4$ is cyclic. What would be a non-cyclic group of order 4? Possible element orders would be 1 or 2. Let a, b be distinct non-identity elements. Some facts

can't happen {

$$\begin{aligned} ab \neq a &\Rightarrow b = e \\ ab \neq b &\Rightarrow a = e \\ ab \neq e &\Rightarrow \underline{a = b^{-1} = b} \end{aligned}$$

so group must consist of $\underbrace{\{e, a, b, ab\}}_{\text{distinct}}$

Note $\underline{ab = (ab)^{-1} = b^{-1}a^{-1} = ba}$ so abelian

write out Cayley Table & compare with

⑥

Cayley Table for $\mathbb{Z}_2 \oplus \mathbb{Z}_2$

	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,1)$	$(0,1)$	$(0,0)$	$(1,1)$	$(1,0)$
$(1,0)$	$(1,0)$	$(1,1)$	$(0,0)$	$(0,1)$
$(1,1)$	$(1,1)$	$(1,0)$	$(0,1)$	$(0,0)$

$\leftarrow \underline{V_4}$