for group: $\phi : G \to \bar{G}$

$\phi(g_1 * g_2) = \phi(g_1) * \phi(g_2)$ Operation Preservation

for rings: $\phi : R \to S$

(i) $\phi(r_1 + r_2) = \phi(r_1) \oplus \phi(r_2)$

$s_1 \oplus s_2$

(ii) $\phi(\underset{R}{r_1 r_2}) = \phi(r_1) \underset{S}{\cdot} \phi(r_2)$

morphism

homomorphism (groups/rings/fields)
isomorphism (bijective)
monomorphism (injective)
epimorphism (surjective)
endomorphism (same ring/field)
automorphism (iso^{m} same object)

(1) $\phi : \mathbb{Z} \to \mathbb{Z}_n$, $k \to k \bmod n$

$$a, b \in \mathbb{Z} \quad \phi(a+b) = \phi(a) \overset{\text{mod } n}{+} \phi(b)$$

$$\phi(ab) = \phi(a) \overset{\text{mod } n}{\cdot} \phi(b)$$

$$a+b = r \bmod n$$

$$n \mid r-(a+b)$$

$$a \equiv \alpha \bmod n \Rightarrow a-\alpha = k_1 n$$
$$b \equiv \beta \bmod n \qquad b-\beta = k_2 n$$

$$a+b-(\alpha+\beta) = (k_1+k_2)n$$

$$a+b \equiv \alpha+\beta \bmod n$$

(2) $* : \mathbb{C} \to \mathbb{C}$ $\quad a+bi \to a-bi$

$$\phi[(a+bi)+(c+di)] = \phi[(a+c)+(b+d)i]$$

$$= (a+c)-(b+d)i = (a-bi)+(c-di)$$

$$\phi(a+bi) + \phi(c+di)$$

$$\phi\left[(a+bi)(c+di)\right] = (ac-bd)+i(bc+ad)$$

$$\downarrow \phi$$

$$\underline{(ac-bd)-i(bc+ad)}$$

$\phi(a+bi) = a-bi \quad \phi(c+di) = c-di.$ $=$

$(a-bi)(c-di) = (ac-bd)-i(bc+ad)$

$\therefore$ conjugation is O.P. for both $+, \cdot$

* $\phi(a+b\sqrt{2}) \rightleftharpoons a-b\sqrt{2}$?

$\phi : \mathbb{R}[x] \to \mathbb{R} \quad f(x) \to f(0)$

$f(x)+g(x) \longmapsto f(0)+g(0)$

$f(x)\cdot g(x) \longmapsto f(0)\cdot g(0)$

R is commutative ring with $\mathrm{char}(R) = 2$

Show $\phi : R \to R \quad a \to a^2$ is o.p.

$a, b \in R$ $\phi(a+b) = (a+b)^2 = a^2 + \underline{2ab} + b^2$

drops

$= a^2 + b^2$

$\phi(a) = a^2$

$\phi(b) = b^2$ $\longrightarrow a^2 + b^2$

$\phi(ab) = a^2b^2$ $\phi(a)\cdot\phi(b) = a^2b^2$

$2\mathbb{Z}$ iso^m to $\mathbb{Z}$ under $+$

$\phi : \mathbb{Z} \to \mathbb{Z}_9$

$$\begin{array}{r} 12578 \\ \times\ 43826 \\ \hline 10058924.0 \end{array} \to 2 \qquad \begin{array}{r} 5 \\ \times\ 5 \\ \hline 25 \end{array} \to 7$$

✱

$$\begin{array}{r} 759 \\ 341 \\ \hline 258{,}819 \end{array}$$

$\boxed{4}\ \boxed{3}\ \boxed{8}\ \boxed{14}$

$\boxed{4}\ \boxed{3}\ \boxed{2}\ \boxed{10}$