2/27

④ Given R comm ring, show char $R[x]$ is same as char R.

Let r be char R. Consider

$$\underbrace{x+x+x\cdots\cdots x}_{\substack{r\\ \text{times}}} = rx$$

$$\nearrow a \in R \qquad \underbrace{ax+ax+\cdots ax} = (\underset{0}{ra})x = 0$$

$$a(rx) = 0$$

⑥ All poly's of degree 2 in $\mathbb{Z}_2[x]$

$x^2, x^2+1, x^2+x, x^2+x+1$

⑪ $\dfrac{x^3+2x+A}{3x+2}$ in $\mathbb{Z}_5$

$$\begin{array}{rl} & 2x^2 \\ 3x+2 \,\big) & \overline{x^3+2x+A} \\ & x^3+ \end{array}$$

⑪

$$2x^2 + 2x + 1$$

$$3x+2 \overline{) x^3 + \_\_ + 2x + 4}$$

$$x^3 + 4x^2$$

$$x^2 + 2x$$

$$x^2 + 4x$$

mod 5

$$+3x + 4$$

$$3x + 2$$

$$\text{Rem} = 2$$

⑦

⑫

$$4x^2 + 3x + 6$$

$$3x^2+2x+1 \overline{) 5x^4 + 3x^3 + \qquad + 1}$$

$$5x^4 + x^3 + 4x^2$$

$$2x^3 + 3x$$

$$3x^2$$

$$2x^3 + 6x^2 + 3x$$

$$4x^2 + 4x + 1$$

$$4x^2 + 5x + 6$$

$$\text{Rem} \quad 6x + 2$$

⑬ Show $2x+1$ has mult inverse in $\mathbb{Z}_4[x]$

$$(2x+1)(2x+1) = 4x^2+4x+1 \equiv 1 \ \underline{\bmod 4}$$

⑭ Are there any non constant poly's in $\mathbb{Z}[x]$ that have inverses?

NO!

⑮ Let p be prime. Are there any non-constant poly's in $\mathbb{Z}_p[x]$ which are invertible.

NO!

⑰ Let D be a domain. $f(x), g(x) \in D[x]$

Prove $\partial f + \partial g = \partial(fg)$.

Let $f(x) = a_n x^n + \cdots + a_1 x + a_0$

$g(x) = b_m x^m + \cdots \ b_1 x + b_0$

$$f(x) \cdot g(x) = a_n b_m \boxed{x^{n+m}} + \cdots + \cdots a_0 b_0$$

⑱ Prove $\langle x\rangle$ in $\mathbb{Q}[x]$ is maximal: /

Construct $\frac{\mathbb{Q}[x]}{\langle x\rangle} \overset{\phi}{\cong} \mathbb{Q}$ ↖ field

Conclude from M.Id.Th w/ $\langle x\rangle$ is max in $\mathbb{Q}[x]$.

⑲ Let F be an infinite field. If $f(a)=0$ for infinitely many elements of F, then $f(x)=0$. Violates # zeroes theorem if $\partial f>0$.

⑳ Let F be an infinite field and suppose $f(x), g(x) \in F[x]$. Show that if $f(a)=g(a)$ for ∞-ly many $a \in F$, then $f(x)=g(x)$. Create $h(x)=f(x)-g(x)$.

㉒ Show $\mathbb{Z}[x]$ is not a PID.

$\langle x\rangle \subsetneq \langle \underline{2,x}\rangle \subsetneq \mathbb{Z}[x]$ Bezout Domain
(2 gen req'd)

Find $f(x) \in \mathbb{Z}[x] \ni \frac{1}{2}, -\frac{1}{3}$ are zeroes

$$\left(x-\frac{1}{2}\right)\left(x+\frac{1}{3}\right) = x^2 - \frac{1}{6}x - \frac{1}{6}$$

$$2\cdot\left(x-\frac{1}{2}\right)\cdot 3\cdot\left(x+\frac{1}{3}\right)$$

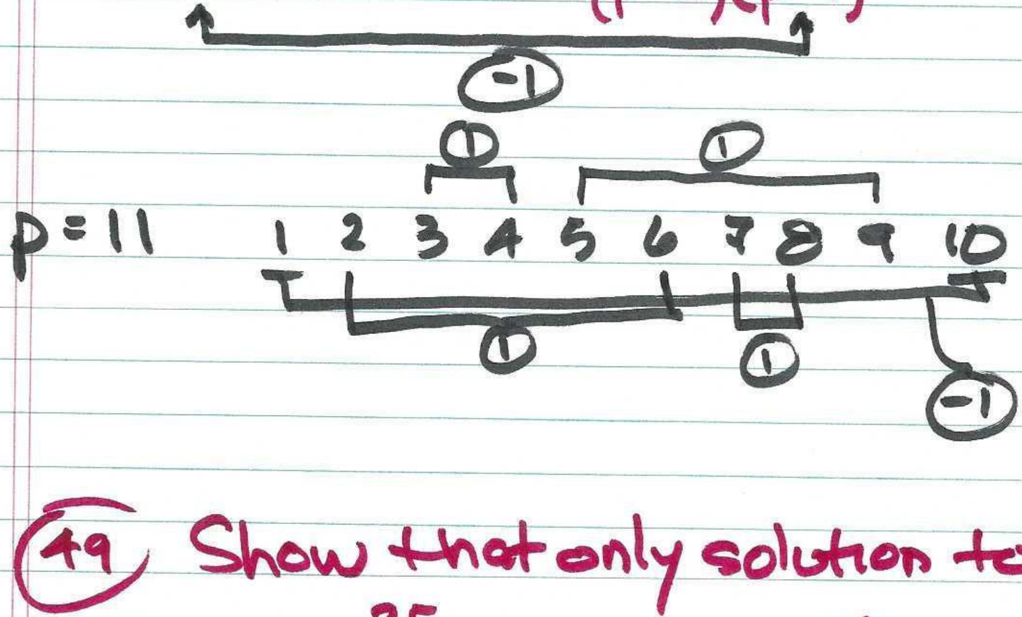
$$(2x-1)(3x+1)$$

㉜ Prove Wilson's thm

If p is prime, then $\underline{\underline{(p-1)! \equiv -1\ (p)}}$

Take p as odd prime

$1\cdot2\cdot3\cdot4\cdots(p-2)(p-1)$



㊾ Show that only solution to

$$x^{25}-1=0 \text{ in } \mathbb{Z}_{37} \text{ is } 1.$$

$a^{25} \equiv 1 \bmod 37$