(1)

$\langle G, * \rangle$

G elements

$\downarrow$ define binary operation (+)

M magma

$\downarrow$ associativity

S semi-group

$\downarrow$ identity

Monoid

$\downarrow$ add inverses

<u>Group</u>

$\downarrow$ commutativity

Abelian Group under +

| add new operation ($\cdot$)

($\cdot$) is associative

$[a*(b*c) \neq (a*b)*c]$

Ring! $\longrightarrow$ distributive: $a(b+c) = ab + ac$

Exemples of Rings:

1. $\mathbb{Z}$ units are ± 1 comm w/ identity
2. $\mathbb{Z}_n$ U_n comm w/ identity
3. → $\mathbb{Z}[x]$ poly's w/ coeff from $\mathbb{Z}$ units ± 1
4. $\mathbb{Z}[[x]]$ formal power series with coeff from $\mathbb{Z}$
5. $M_2(\mathbb{Z})$ not comm but has identity

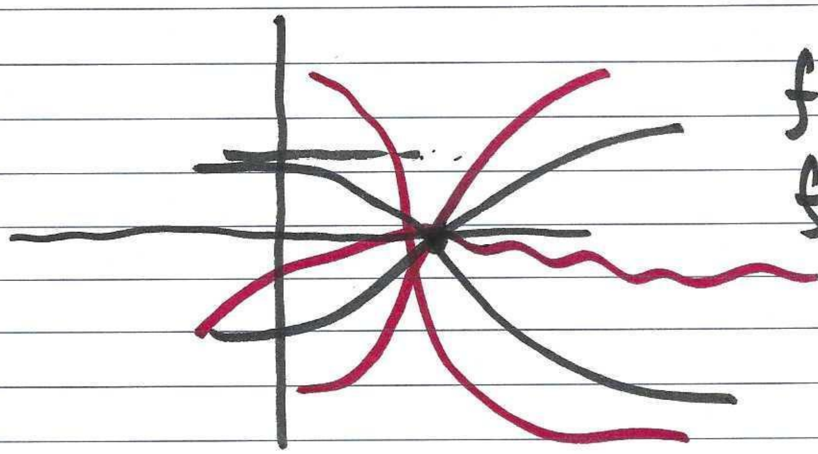
$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 9 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & \\ & \end{bmatrix}$$

6. $n\mathbb{Z} \rightsquigarrow 2\mathbb{Z}$ $2\alpha + 2\beta = 2(\alpha + \beta)$

commutative non-identity

7



$f(x)+g(x) = (f+g)(x)$

$f(x)\cdot g(x) = (fg)(x)$

8 Given $\{R_1, R_2, \cdots, R_n\}$

$R_1 \oplus R_2 \oplus \cdots \oplus R_n$

elements $(r_1, r_2, \cdots, r_n)$

where $r_i \in R_i$

Rules for Multiplication

1) $a\cdot 0 = 0\cdot a = 0$
2) $a(-b) = (-a)b = -(ab)$
3) $(-a)(-b) = ab$
4) $a(b-c) = ab-ac$

   $(b-c)a = ba-ca$

If $1 \in R$

5) $(-1)a = -a$

6) $(-1)(-1) = 1$

Thm $1 \in R$ & $1' \in R \rightarrow 1 = 1'$

Pf: $\underline{1' = 11' = 1}$

No-no's:

In a ring, if $a^2 = a$

$ab \neq ac$ no cancellation in rings.

Subgroup test $G \supset S$

→ If $\forall a, b \in S$, $ab^{-1} \in S$ then $S \leq G$

So Subring Test: $\overset{\text{ring}}{R} \supset \overset{\text{set}}{S}$

1) Show S is abelian subgp

2) Show $(\cdot)$ <u>closed</u>

1) Is $\mathbb{Z}[i]$ subring of $\mathbb{C}$

$$(a+bi)-(c+di) = \underline{(a-c)+i(b-d)}$$

$$(a+bi)(c+di) = (\underline{ac-bd})+i(\underline{ad+bc})$$

2) $\{0\}$ subring, R is subring of itself