SUMMARY OF RING INFO	Ring	Domain	Euclidean Domain	Principal Ideal Domain	Unique Factorization Domain	Field
Definition	abelian group under ⊕ with associative & distributive ⊗	ring with commutative ⊗, identity, no zero divisors, 0≠1	domain with norm & euclidean algorithm	domain with all ideals principal	domain with unique factorization into irreducibles	domain with inverses for ⊗ except "0"
Examples	**Z**, GL(n,**Z**)	**Z**, **Z**[i]	**Z**, **Z**[x]	**Z**	**Z**, **k**[x], **k** field	**Q**, **R**, **C**, **Z**(p)
Forward Implications	no	no	euclidean implies principal	principal implies unique factorization	no	***
Backward Implications	***	domain is ring	ED is domain	PID only implies domain	UFD only implies domain	field implies all prior
Counterexample	***	Z[√-5] is domain only	***	**Z**[1/2(1+√-19)] not ED / **Z** not field	**Z**[x] not PID, hence not ED	
Norm?	no, but could be imposed	no	by definition	Dedekind-Hasse norm implied		
Ideals	chirality possible	bilateral	bilateral	bilateral	bilateral	only {0} and improper
Prime Ideals	commutative, unital ring mod prime ideal is domain	maximal ideal always prime / if all prime ideals principal, then PID	nonzero prime ideals and maximal ideals are the same	nonzero prime ideals and maximal ideals are the same		{0} only prime ideal

Maximal Ideals	commutative, unital ring mod maximal ideal is field					{0} only maximal ideal
Prime Elements (nonzero nonunits)	primes defined for commutative, unital rings	prime element always irreducible	primes and irreducibles are the same	primes and irreducibles are the same	primes and irreducibles are the same	primes and irreducibles are the same
Counterexample	no implication between primes and irreducibles	3 irreducible in Z[√-5] but not prime	***	***	***	***
Irreducibles (nonzero nonunits)	irreducibles defined for commutative, unital rings	irreducibles need not be primes	primes and irreducibles are the same	primes and irreducibles are the same	primes and irreducibles are the same	primes and irreducibles are the same
Polynomial Ring over...	is ring but need not be domain / if PID and ring commutative, then ring is field	is domain	is not necessarily ED	is not necessarily PID, but is if coeff are field / noetherian	is UFD and vice versa	is UFD
Ideal in Polynomial Ring over...				prime if coeff from prime ideal		
Chain Conditions			noetherian	noetherian		