

TB Proposition 1: There exists a unique set with no elements.

TB Proposition 2: (Existence of intersection) If P and Q are sets, then there is a set R such that $x \in R$ if and only if $x \in P$ and $x \in Q$.

TB Proposition 3: Given A , there is a unique B such that $x \in B$ if and only if $x \in A$ and $P(x)$ is true.

TB Proposition 4: For any property P , $\{x \in \emptyset | P(x) \text{ is true}\} = \emptyset$.

TB Proposition 5: The set given by the Axiom of Pair is unique.

TB Proposition 6: The set given by the Axiom of Union is unique.

TB Proposition 7: The set given by the Axiom of Power Set is unique.

TB Proposition 8: The set of all x such that $x \in A$ and $x \notin B$ exists.

TB Proposition 9: The Weak Axiom of Existence (some set exists) implies the Axiom of Existence.

TB Proposition 10: $\emptyset(X) \subseteq X$ is never true.

TB Proposition 11: Given the set A , the set $\{x | x \notin A\}$ does not exist.

DS Proposition 12: Given the nonempty system of sets S , $\bigcap S$ exists.

NS Proposition 13: $\bigcap \emptyset$ is the universe.

MJ Proposition 14: Given a set S , $\emptyset(S)$ is a commutative ring with identity under the operations Δ and \cap

LB Proposition 15: $\bigcup S^c = (\bigcap S)^c$

LB Proposition 16: $\bigcap S^c = (\bigcup S)^c$

NS Proposition 17: Let R be a binary relation, both $domR$ and $ranR$ exist.

JW Proposition 18: If $a \in A$ and $b \in A$, then $(a, b) \in 2^{2^A}$.

JW Proposition 19: $(a, b) = (b, a)$ implies $a = b$.

NS Proposition 20: If S and R are relations on A , then $S \circ R$ exists.

NS Proposition 21: The following are true for relation R and sets $A, B \subseteq domR$: (i) $R(A \cup B) = R(A) \cup R(B)$, (ii) $R(A \cap B) \subseteq R(A) \cap R(B)$, (iii) $R(A - B) \supseteq R(A) - R(B)$.

DS Proposition 22: Composition of relations on a given set is associative.

JW Proposition 23: $\exists X$ such that $X^3 \neq X \times X^2$

TB Proposition 24: $A \times B = \emptyset$ if and only if at least one of A and B is empty.

NS Proposition 25: If f is a function, the following are true: (i) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, (ii) $f^{-1}(A \cap B) \subseteq f^{-1}(A) \cap f^{-1}(B)$, (iii) $f^{-1}(A - B) \supseteq f^{-1}(A) - f^{-1}(B)$.

TB Proposition 26: Every system of sets S can be indexed by a function $f: I \rightarrow S$, where I is a suitable index set

LB Proposition 27: $\bigcup_{\alpha \in I} S_\alpha = \bigcup_{C \in S} (\bigcup_{\alpha \in C} F_\alpha)$

LB Proposition 28: $\bigcap_{\alpha \in I} S_\alpha = \bigcap_{C \in S} (\bigcap_{\alpha \in C} F_\alpha)$

LB Proposition 29: If f is a function, $f(\bigcup_{\alpha \in A} F_\alpha) = \bigcup_{\alpha \in A} f(F_\alpha)$

LB Proposition 30: If f is a function, $f(\bigcap_{\alpha \in A} F_\alpha) \subseteq \bigcap_{\alpha \in A} f(F_\alpha)$

MJ Proposition 31: The composition of injective functions is injective and the composition of surjective functions is surjective.

NS, MJ Proposition 32: If $F: X \rightarrow X$ is injective (resp. surjective) then it is surjective (resp. injective) provided _____.

DS Proposition 33: There is no set that contains all sets equipotent to the set $A \neq \emptyset$.

Hint: Russell paradox.

Proposition 34: Let $S(x)$ be the successor of x . There does not exist z such that $x \subset z \subset S(x)$.

Proposition 35: Let $P(x)$ be a property (possibly with parameters). If $P(0)$ is true and for all n , $P(n)$ implies $P(n+1)$, then $P(n)$ is true for all $n \in \mathbb{N}$. Can't assume \mathbb{N} is well-ordered..yet.

Proposition 36: Let $P(x)$ be a property (possibly with parameters). Assume that for all $n \in \mathbb{N}$, if $P(k)$ holds for $k < n$, then $P(n)$ is true. Then $P(n)$ is true for all $n \in \mathbb{N}$.

Proposition 37: Let $P(x,y)$ be a property. If $P(k,l)$ holds for all $k,l \in \mathbb{N}$ such that $k < m$ or ($k = m$ and $l < n$) implies $P(m,n)$ is true, then $P(m,n)$ is true for all $m,n \in \mathbb{N}$.

Proposition 38: Let $P(x)$ be a property (possibly with parameters). If $P(k)$ is true and for all $n > k$, $P(n)$ implies $P(n+1)$, then $P(n)$ is true for all $n > k$.

Proposition 39: Assuming \mathbb{N} is well ordered by $<$, let $P(x)$ be a property (possibly with parameters). If $P(0)$ is true and for all n , $P(n)$ implies $P(n+1)$, then $P(n)$ is true for all $n \in \mathbb{N}$.

Proposition 40: Every well-ordered set is linearly ordered.

Proposition 41: \mathbb{N} is well ordered by $<$.

Proposition 42: There is no function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $f(n) > f(n+1)$.

Proposition 43: \mathbb{N} is equipotent to infinitely many of its proper subsets.

Proposition 44: Given $n \in \mathbb{N}$, there is no $k \in \mathbb{N}$ such that $n < k < n+1$.

Proposition 45: For all $m,n \in \mathbb{N}$, if $m < n$, then $m+1 < n$. Hint: use Proposition 44.

Proposition 46: The successor mapping $S(n)$ is injective.

You may assume ..Theorem: Let $a : P \rightarrow A$ and $g : P \times A \times \mathbb{N}$ be functions. There exists a unique function $f : P \times \mathbb{N} \rightarrow A$ such that (i) $f(p,0) = a(p)$ for all $p \in P$ and (ii) $f(p,n+1) = g(p,f(p),n)$ for all $n \in \mathbb{N}$ and $p \in P$.

Proposition 47: There is a unique binary operation $+$: $\mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ such that (i) $+(m,0) = m$ for all $m \in \mathbb{N}$ and (ii) $+(m,n+1) = +(m,n) + 1$ for all $m,n \in \mathbb{N}$

Proposition 48: The operation of addition defined in Prop. 47 is commutative.

Proposition 49: (Refer to the Peano Axioms in the ZFC summary) \mathbb{N} satisfies the Peano axioms.

Proposition 50: If $m,n \in \mathbb{N}$, then $m \leq n$ if and only if there exists a unique $k \in \mathbb{N}$ such that $n = m + k$. Define $-(n,m) = k$.

Proposition 51: There exists a unique binary operation \cdot : $\mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ such that $m \cdot 0 = 0$ for all $m \in \mathbb{N}$ and $m \cdot (n+1) = m \cdot n + m$ for all $m,n \in \mathbb{N}$. Recall: addition is now defined.

Proposition 52: Multiplication (the operation in Prop. 51) is commutative.

Proposition 53: Define $E : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ as follows: (i) $E(m,0) = 1$, for all $m \in \mathbb{N}$ (even 0!..don't tell anyone), (ii) $E(m,n+1) = m \cdot E(m,n)$. Prove the usual laws of exponents.

Proposition 54: Given a set of n -tuples $R \subseteq X^n$ for some nonempty set X , show that R induces a relation with arity n .

Proposition 55: The structure $(\mathbb{N}, <)$ has only one automorphism.

Proposition 56: The structure $(\mathbb{Z}, <)$ has infinitely many automorphisms.

Transfinite Arithmetic:

Proposition 57: Cardinal addition is commutative.

Proposition 58: Cardinal addition is associative.

Proposition 59: Cardinal multiplication is commutative.

Proposition 60: Cardinal multiplication is associative.

ND Proposition 61: Cardinal multiplication distributes over cardinal addition.

NS Proposition 62: $\aleph_0 + \aleph_0 = \aleph_0$ and $\aleph_0 \cdot \aleph_0 = \aleph_0$

NS Proposition 63: $\aleph_0 + n = \aleph_0$ and $\aleph_0 \cdot n = \aleph_0$ for any $n \in \mathbb{N}$

TB Proposition 64: $\aleph_0 + \aleph_1 = \aleph_1$ and $\aleph_0 \cdot \aleph_1 = \aleph_1$

Proposition 65: For cardinals κ, λ, μ it is true that $\kappa^{\lambda+\mu} = \kappa^\lambda \cdot \kappa^\mu$

Proposition 66: For cardinals κ, λ, μ it is true that $(\kappa^\lambda)^\mu = \kappa^{\lambda\mu}$

Proposition 67: $\aleph_0^{\aleph_0} > \aleph_0$

Proposition 68: Show that every $n \in \mathbb{N}$ is an initial ordinal

Proposition 69: Every well-orderable set is equipollent to an ordinal number

Proposition 70: Every well-orderable set is equipollent to a unique initial ordinal number

Proposition 71: The Hartogs number for any set is an initial ordinal

Proposition 72: For any ordinal β , $\beta \cdot 1 = \beta$

Proposition 73: For any ordinal β , $\beta \cdot 2 = \beta + \beta$

Proposition 74: $2 \cdot \omega \neq \omega \cdot 2$

Proposition 75: $2 \cdot \omega = \omega$

Proposition 76: For any ordinal β , $\beta^2 = \beta \cdot \beta$

Proposition 77: $1^\omega = 1$

Proposition 78: $2^\omega = 3^\omega$

Proposition 79: $\omega^\omega > \omega$

Proposition 80: Find the least ordinal ξ such that $\omega + \xi = \xi$

Proposition 81: Every strong limit cardinal is a limit cardinal

Proposition 82: Every limit cardinal is a strong limit cardinal if GCH holds

Proposition 83: Every successor cardinal $\aleph_{\alpha+1}$ is regular

Proposition 84: There exist arbitrarily large singular cardinals

Proposition 85: There are arbitrarily large singular cardinals \aleph_α such that $\aleph_\alpha = \alpha$

Proposition 86: The cofinality of α is a limit ordinal

Proposition 87: An ordinal is never less than its cofinality.

Proposition 88: (Test for singularity) A cardinal \aleph_α is singular if $cf(\omega_\alpha) < \omega_\alpha$ and regular if

$cf(\omega_\alpha) = \omega_\alpha$