Let $\underline{U}$ be all elements (a set)

Then 2^U is power set (by axiom)

But $\underline{2^U}$ is set of all sets

So $\Rightarrow\Leftarrow$ via Russell.

Thm/ The collection of ordinals is not a set.

Pf: If it were a set, Say Θ is set ordinals, then $\{\Theta, \cup\Theta\}$ is an ordinal (by construction) and is not in Θ.

Thm/ The collection of sets which are equipotent to a given set A is not a set. (It's a class).

Pf: Consider the collection of one-element sets. Claim: it's a proper class. Suppose not. Form $\underline{\bigcup\{x\}} = V$ (universe)

Form $2^{\overline{\bigcup\{x\}}}$ = "set of all sets".

But Russell paradox says no!

Consider sets of form $\{x\} \times A$

$= \{(x, a_i) \mid x \in \{x\}, a_i \in A \; \exists i \in \Delta\}$

Bijection $A \mapsto \{x\} \times A$ by saying

$a_i \mapsto (x, a_i)$

$\therefore$ card A = card $\underline{\{x\} \times A}$ ■

$\mathbb{R}^{\mathbb{N}}$ Since

$|\mathbb{R}^{\mathbb{N}}| > |2^{\mathbb{N}}| = \aleph_1$

Prop 65: For cardinals κ, λ, μ it is true that $\kappa^{\lambda+\mu} = \kappa^{\lambda} \cdot \kappa^{\mu}$

Let $|A| = \kappa$
$|B| = \lambda$
$|C| = \mu$

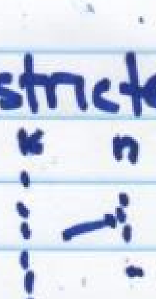
$\kappa^{\lambda+\mu} = |\text{set of functions } B \cup C \text{ to } A|$

Finite: $|A| = 3$
$|B| = 5$
$|C| = 7$

$B \cup C$ has 12 elements

want # unrestricted maps from 12-set to 3-set

3^{12}



n^k fcns

Separately: $\overset{5}{B} \to \overset{3}{A}$ $\quad \overset{7}{C} \to \overset{3}{A}$

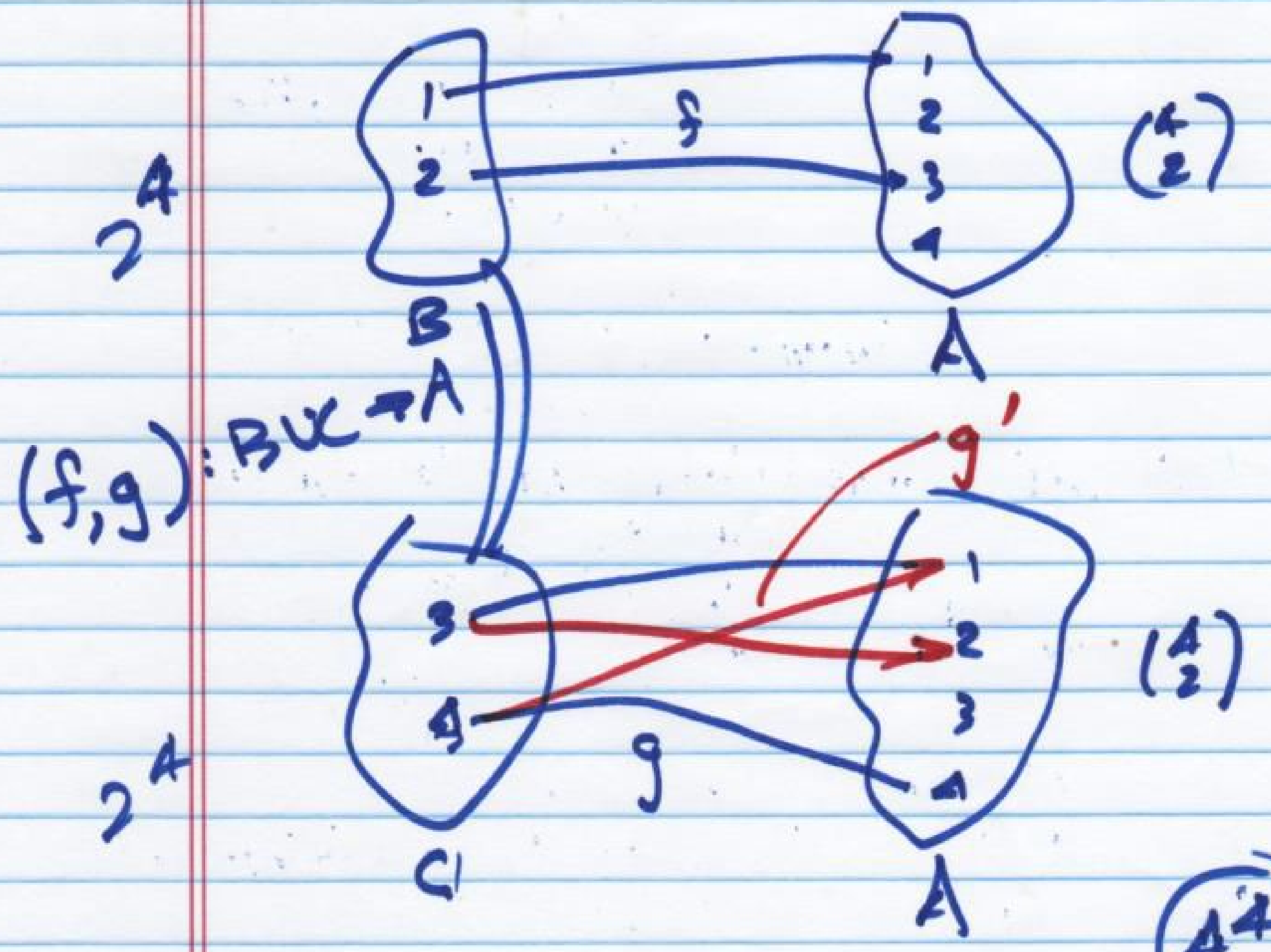
$3^5 \cdot 3^7 = 3^{12}$

4

$K^{\lambda} = \{\text{set of fcns from } B \text{ to } A\}$

$K^{\mu} = \{\text{set of fcns from } C \text{ to } A\}$

$K^{\lambda} \cdot K^{\mu} = \underline{\{(f_{\alpha}, f_{\beta}) \mid f_{\alpha} \in K^{\lambda}_{2},\ f_{\beta} \in K^{\mu}_{2}\}}$



$\binom{4}{2}\binom{4}{2} =$

4^4

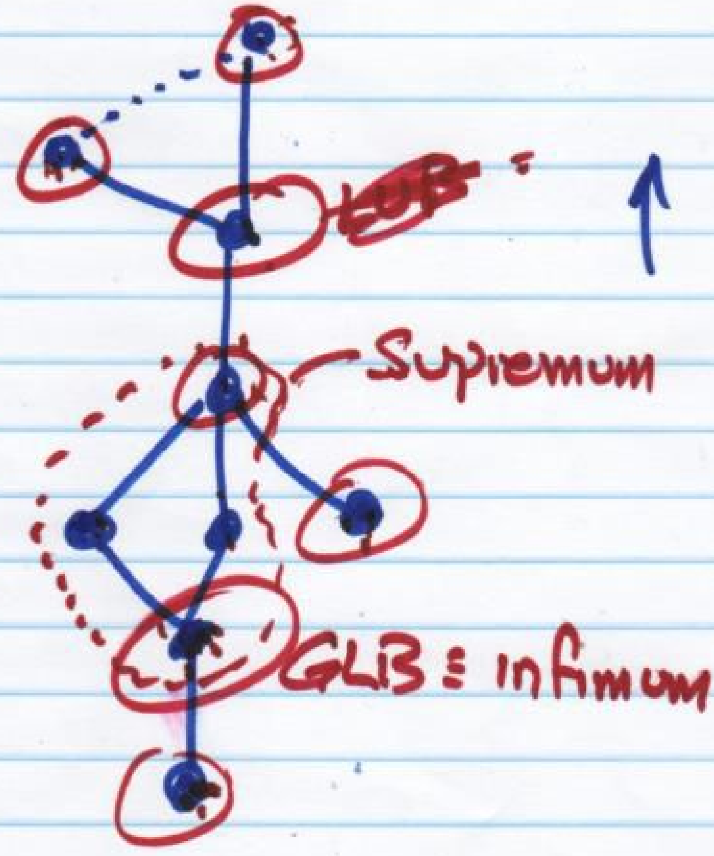
Thm/ If S is well-ordered by $\preceq$

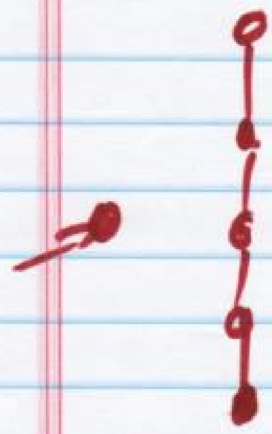
Given $a, b \in S$ then $\{a,b\} \neq \emptyset$

So $\{a,b\}$ has <u>least</u> element, say

a i.e. $a \preceq b$.

P.O.SET
<u>W.O.SET</u>
T.O.SET





Prop 42 $\nexists f : \mathbb{N} \to \mathbb{N} \ni$

$$f(n) > f(n+1)$$

Pf: