

define:

$$H_n(a, b) = \begin{cases} b+1 & \text{if } n=0 \\ a & \text{if } n=1 \text{ ; } b=0 \\ 0 & \text{if } n=2 \text{ ; } b=0 \\ 1 & \text{if } n \geq 3 \text{ ; } b=0 \\ H_{n-1}(a, H_n(a, b-1)) & \text{otherwise} \end{cases}$$

 $n \downarrow$

$$0 \quad H_0(a, b) = b+1 \quad (\text{successor of } b)$$

$$1 \quad H_1(a, b) = H_0(a, H_1(a, b-1))$$

$$= H_0(a, \underbrace{b}_{\text{"b"}} + 1)$$

$$= H_0(a, H_1(a, b-2)) + 1$$

$$= H_1(a, b-2) + 2$$

⋮

$$= H_1(a, 0) + b$$

$$H_1(a, b) = \underline{a + b} \quad \underline{\text{addition}}$$

(2)

$$n=2 \quad H_2(a, b) = H_1(a, H_2(b, b-1))$$

$$H_2(a, b) = a \cdot b \quad \begin{array}{l} \text{multiplication} \\ \underbrace{a + a + \dots + a}_{b \text{ copies}} \end{array}$$

$$H_3(a, b) = a^b \quad \begin{array}{l} \text{exponentiation} \\ \underbrace{a \cdot a \cdot a \dots a}_{b \text{ copies}} \end{array}$$

$$H_4(a, b) = {}^b a = a^{a^{a^{\dots^a}}} \quad \begin{array}{l} \text{(tetration)} \\ \underbrace{\phantom{a^{a^{a^{\dots^a}}}}}_{b \text{ copies}} \end{array}$$

Ex: $2^{(2^{(2^2)})} = ? \quad 2^{2^4} = 2^{16} =$

not $4^2 = 16^2 = 256 \quad \text{NO}$

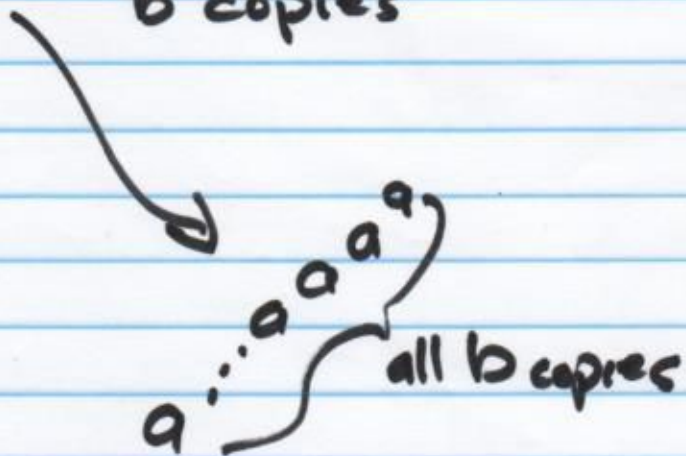
(3)

$$a \uparrow b \equiv a^b$$

$$a \uparrow \uparrow b \text{ or } a \uparrow^{(2)} b \equiv b^a$$

$$a \uparrow \uparrow \uparrow b \text{ or } a \uparrow^{(3)} b$$

$$a \uparrow \uparrow b = \underbrace{a \uparrow a \uparrow a \uparrow \dots (a \uparrow a)}_{b \text{ copies}}$$



$$a \uparrow \uparrow \uparrow b = a \uparrow \uparrow a \uparrow \uparrow a \uparrow \uparrow \dots (a \uparrow \uparrow a)$$

④

Graham's Number g_n ↘

$$= \begin{cases} 3 \uparrow \uparrow \uparrow 3 & \text{if } n=1 \\ 3 \uparrow g_{n-1} 3 & \text{if } n \geq 2 \end{cases}$$

$$3 \uparrow \uparrow \uparrow 3 = \underline{3 \uparrow \uparrow (3 \uparrow \uparrow 3)}$$

$$V_{\text{Planck}} = \sqrt{\frac{(\hbar G)^3}{c^9}} \sim 10^{-105} \text{ m}^3$$

$$\rightarrow \lambda'_\alpha + \lambda'_\beta = \lambda'_{[\max(\alpha, \beta)]} \quad \textcircled{5}$$

Prop 62

$$\lambda'_0 + \lambda'_0 = \lambda'_0$$

$$\lambda'_0 \cdot \lambda'_0 = \lambda'_0$$

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