define:

$$H_n(a,b) = \begin{cases} b+1 & \text{if } n=0 \\ a & \text{if } n=1 \text{ \& } b=0 \\ 0 & \text{if } n=2 \text{ \& } b=0 \\ 1 & \text{if } n\geq 3 \text{ \& } b=0 \\ H_{n-1}(a, H_n(a,b-1)) & \text{otherwise} \end{cases}$$

$n \downarrow$

0 $\quad H_0(a,b) = b+1 \quad$ (<u>successor</u> of b)

1 $\quad H_1(a,b) = H_0(a, H_1(a,b-1)$

$$= \underbrace{H_1(a,b-1)}_{\text{"b"}} + 1$$

$$= H_0(a, H_1(a,b-2)) + 1$$

$$= H_1(a,b-2) + 2$$

$$\vdots$$

$$= H_1(a,0) + b$$

$$H_1(a,b) = \underline{a+b} \quad \underline{\text{addition}}$$

n = 2 $H_2(a,b) = H_1(a, H_2(a,b-1))$

$\vdots$

$H_2(a,b) = a \cdot b$ multiplication $\underbrace{a+a+\cdots+a}_{b \text{ copies}}$

$H_3(a,b) = a^b$ exponentiation $\underbrace{a\cdot a\cdot a\cdots a}_{b \text{ copies}}$

$H_4(a,b) = {}^{b}a = \left.a^{a^{a^{\cdot^{\cdot^{\cdot^{a}}}}}}\right\} b \text{ copies}$ (tetration)

Ex: $2^{(2^{(2^{2})})} = ?$ $2^{2^{4}} = 2^{16} =$

not $4^{2^{2}} = 16^{2} = 256$ NO

③

$a \uparrow b \equiv a^b$

$a \uparrow\uparrow b$ or $a \uparrow^{(2)} b \equiv {}^b a$

$a \uparrow\uparrow\uparrow b$ or $a \uparrow^{(3)} b$

$$a \uparrow\uparrow b = \underbrace{a \uparrow a \uparrow a \uparrow (\cdots(\cdot a \uparrow a))}_{b \text{ copies}}$$

$$a^{\cdot^{\cdot^{\cdot^{a^{a^{a^{a}}}}}}}\Big\}\text{all } b \text{ copies}$$

$$a \uparrow\uparrow\uparrow b = a \uparrow\uparrow a \uparrow\uparrow a \uparrow\uparrow \cdot (\cdot \cdot (a \uparrow\uparrow \cdot a))$$

4

$$\underline{\text{Graham's Number}} \overset{g_n}{\searrow} = \begin{cases} 3\uparrow\uparrow\uparrow\uparrow 3 & \text{if } n=1 \\ 3\uparrow^{g_{n-1}} 3 & \text{if } n\geq 2 \end{cases}$$

$$3\uparrow\uparrow\uparrow\uparrow 3 = \underline{3\uparrow\uparrow\uparrow}(\underline{3\uparrow\uparrow\uparrow 3})$$

$$V_{planck} = \sqrt{\frac{(\hbar G)^3}{c^9}} \sim 10^{-105}\ \underline{m}^3$$

$$\rightarrow \lambda'_\alpha \dot{+} \lambda'_\beta = \lambda'_{[\max(\alpha,\beta)]}$$

Prop 62 $\lambda'_0 + \lambda'_0 = \lambda'_0$ NS

$\lambda'_0 \cdot \lambda'_0 = \lambda'_0$