Inductive Set: Let I be an ind. set

then: (i) $\emptyset \in I$

(ii) If $A \in I$ then $\underline{A \cup \{A\}} \in I$

$\emptyset \in I$ $\quad$ $0 \leftarrow \emptyset$

$\emptyset \cup \{\emptyset\}$ $\quad$ $1 \leftarrow \{\emptyset\}$

$[\emptyset \cup \{\emptyset\}] \cup \{\emptyset \cup \{\emptyset\}\}$ $\quad$ $2 \leftarrow \{\emptyset, \{\emptyset\}\}$

$\vdots$

$3 \leftarrow \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

$\vdots$

ord card

$|\mathbb{N}| = \aleph_0$ $\quad$ $\underbrace{\omega = \mathbb{N}}_{\aleph_0}$ these are the ordinal numbers

<u>Ord</u>

$|2^{\mathbb{N}}| > \aleph_0$

$= \aleph_1$

$\underline{|2^A| > |A|}$ <u>Cantor</u>

finite

②

$\cdots \omega < \omega+1 < \omega+2$

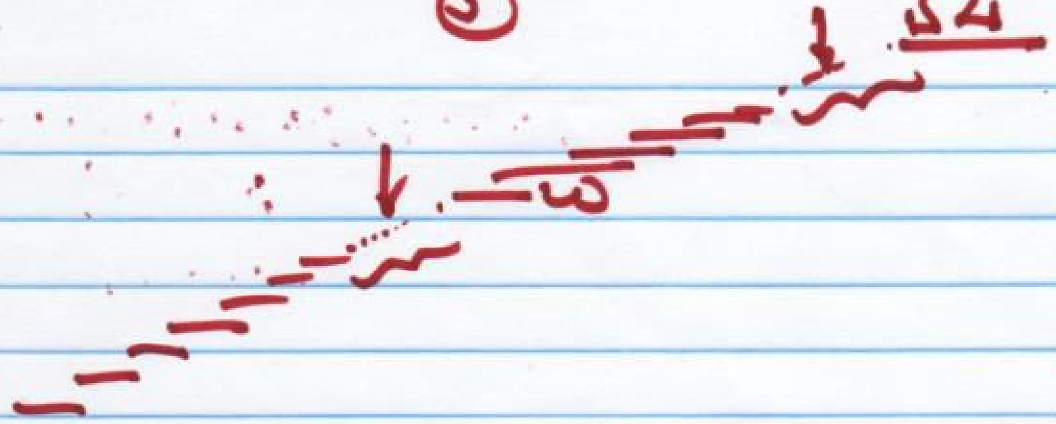
bijection

ω		$\omega+1$
1	$\rightarrow$	$\omega+1$
2	$\rightarrow$	1
3	$\rightarrow$	2
4	$\rightarrow$	3
$\vdots$		$\vdots$

1	$\rightarrow$	$\omega+1$
2	$\rightarrow$	ω
3	$\rightarrow$	1
4	$\rightarrow$	2
5	$\rightarrow$	3
$\vdots$		$\vdots$

1	$\rightarrow$	$\omega+1$
2	$\rightarrow$	1
3	$\rightarrow$	2
$\vdots$		$\vdots$
n	$\rightarrow$	$n-1$
ω	$\rightarrow$	ω

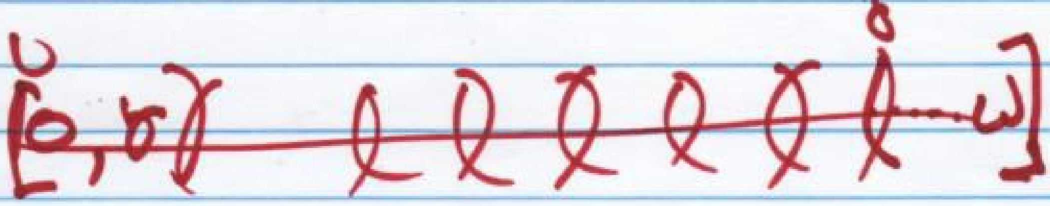
3

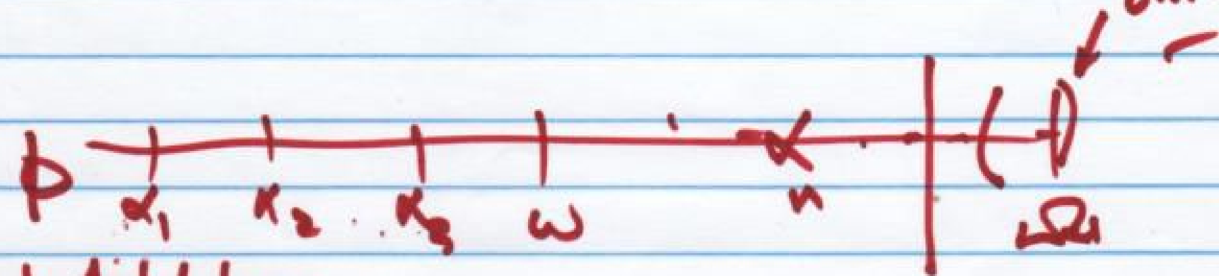


$[0, \omega]$

(α, β)

$[0, \gamma)$ $(\delta, \omega]$





Cardinal Addition

$\kappa_1 + \kappa_2 = \text{card}(A \cup B)$

where $\text{card } A = \kappa_1$, $\text{card } B = \kappa_2$

$A \cap B = \emptyset$

$\kappa_1 \cdot \kappa_2 = \text{card}(A \times B)$ where

A & B are as before

$$\aleph_\alpha \cdot \aleph_\beta = \aleph_{[\alpha,\beta]}$$

exponentiation:

$\kappa_1^{\kappa_2}$ count fcns from A to B

where $\text{card}\, A = \kappa_2$ $\text{card}\, B = \kappa_1$