~~HW~~ C 1-17-18

1 ~~MATH~~

Prop. 15: $\bigcup S^c = (\bigcap S)^c$

Prop. 7: Pf: Given S, form power set(s) P_1, P_2

If $P_1 \neq P_2$ (check via Ext.)

Suppose $A \in P_1$ but $A \notin P_2$

But $A \subseteq S$ so $A \in P_1$, but $A \notin P_2 \Rightarrow P_2$ is not powerset $\rightarrow\leftarrow$ $\square$

~~Thm~~

Given sets A, B denote cardinality by $|\cdot|$

1) If $|A| \leq |B|$ must have $f: A \to B$ f is injective

2) If $|A| \geq |B|$ must have $f: A \to B$ ~~f~~ g is surjective

3) If $|A| = |B|$, must have $\varphi: A \to B$ φ is bijection

Cantor-Shröder-Bernstein Thm

$\hookrightarrow f: A \to B$ inj. $\wedge$ $g: B \to A$ inj $\Rightarrow |A| = |B|$

Fun Fact: $|X| < |2^X|$

Pf: Assume $|X| = |2^X|$

$\Rightarrow \exists \varphi: X \to 2^X$ s.t φ is bijection

Call $a \in X$ "good" if $a \in \varphi(a)$

Call $a \in X$ "bad" if $a \notin \varphi(a)$

Let $B = \{a \in X : a \notin \varphi(a)\}$

So $B \in 2^X$. Let $b = \varphi^{-1}(B)$

② 1-17-18

Two cases:

(i) b is good: $\Rightarrow b \in \varphi(b) = B$ $\Rightarrow\Leftarrow$

(ii) b is bad $\Rightarrow b \notin \varphi(b) = B$ $\Rightarrow\Leftarrow$

$\therefore \nexists$ bijection: $|X| \to |2^X|$ $\square$

$|\mathbb{N}| < |2^{\mathbb{N}}| \longrightarrow \aleph_0 < \aleph_1$

$|\mathbb{R}| < |2^{\mathbb{R}}|$

<u>Infinitesimals</u>

$f(x) = x^3$

$$\lim_{\Delta x \to 0} \left(\frac{(x+\Delta x)^3 - x^3}{\Delta x} \right) = \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2 \right)$$

$$\lim_{\Delta x \to 0} \left(\frac{(\Delta x)^2}{\Delta x} \right) = \lim_{\Delta x \to 0} (\Delta x) = 0$$

<u>Filter (of sets)</u>

(nonvoid coll. of nonvoid)

Given a space X, a filter (on X) $\mathcal{F}$ is a family of subsets of X with 2 properties

i) If $A, B \in \mathcal{F}$, then $A \cap B \in \mathcal{F}$

ii) $A \subseteq B$ and $A \in \mathcal{F} \Rightarrow B \in \mathcal{F}$

$\xrightarrow{\text{tail}}$

$1, 2, 3, [4, \ldots$

Fréchet filter

Prop. 12 → David

Prop. 13 → Nick

Prop. 14 → Mike

15, 16 → Leigh