

TABLE OF INTEGER PARTITION NUMBERS

<u>k</u>	<i>← Maximum number of parts →</i>										
<u>0</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>
0	1	1	1	1	1	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1	1	1	1
2	0	1	2	2	2	2	2	2	2	2	2
3	0	1	2	3	3	3	3	3	3	3	3
4	0	1	3	4	5	5	5	5	5	5	5
5	0	1	3	5	6	7	7	7	7	7	7
6	0	1	4	7	9	10	11	11	11	11	11
7	0	1	4	8	11	13	14	15	15	15	15
8	0	1	5	10	15	18	20	21	22	22	22
9	0	1	5	12	18	23	26	28	29	30	30
10	0	1	6	14	23	30	35	38	40	41	42

NB: The (k,k) entries in the table are actually the statistic that appears in the fourth row first column of the Twelve-Fold Way. The entries then stabilize going to the right, as the maximum number of partitions has been reached.

To find $\mathbf{p}_n(\mathbf{k})$, the number of partitions of k with **exactly** n parts, find table entries (k,n) - (k,n - 1) and take their difference. Be careful reading the literature...some authors use $\mathbf{p}_n(\mathbf{k})$ to mean the numbers in the table above. Our notation conforms to the meaning of $\mathbf{p}_n(\mathbf{k})$ in the Twelve-Fold Way.

For example, $\mathbf{p}_4(\mathbf{5})$, the number of partitions of 5 with exactly 4 parts is 6 - 5 = 1.

Ex: To distribute 10 candy bars (indistinct) to 6 children (indistinct) so that each child gets at least one candy bar, we want the fourth row, third column of the Twelve-Fold Way. This asks for the number of surjective maps of an indistinct 10-set to an indistinct 6-set. We find the (10,6) and (10,5) entries in the table above and take their difference: 5. Evidently there are 35 partitions of 10 which have at most 6 parts, and 30 which have at most 5 parts. The difference is those with exactly 6 parts.

The bijective correspondence between partitions with at most n parts and partitions with largest part equal to n, which we established using conjugate Ferrers graphs, is crucial in going from the generating function coefficients to the entries in the table above.