*Hypermath/Calculus*

## PARAMETRIC INTEGRATION

BACKGROUND:

Some superficially difficult integrals can be done by introducing an arbitrary parameter into the problem, manipulating the parameter to do the integral, then removing the parameter by setting it equal to 1.

TYPICAL PROBLEM:

Suppose we want to evaluate $\int xe^x dx$. (We could use integration by parts for this one, but this illustrates the method). Note first that $\frac{d}{dt}e^{tx} = xe^{tx}$. Now define $I(t) = \int e^{tx}dx = \frac{e^{tx}}{t} + Ct + D$, where $C$ and $D$ are constants. Then $I'(t) = \frac{d}{dt}\int e^{tx}dx = \frac{d}{dt}\left(\frac{e^{tx}}{t} + Ct + D\right) = \frac{txe^{tx} - (1)e^{tx}}{t^2} + C$. But then $\frac{d}{dt}\int e^{tx}dx = \int \frac{\partial}{\partial t}e^{tx}dx = \int xe^{tx}dx = \frac{txe^{tx} - (1)e^{tx}}{t^2} + C$. Now set $t = 1$, and $\int xe^x dx = e^x(x-1) + C$.