① $x^3 + \alpha x^2 + \beta x + \gamma$

$\alpha, \beta, \gamma \in \{0, 1, 2\}$

$(x - r_1)(x - r_2)(x - r_3)$

27 total
16 factor
11 irred

~~3 way all diff~~ $(x - r_1)(x^2 + \mu x + \lambda)$

3 choices · 9 choices

splits
- 1 way all diff
- 3 ways to pick r_1 then 2 ways r_2, r_3 same
- 3 ways all same

linear × irred quad

$x^2 + \alpha x + \beta$

9 quad

3 choices for linear factor

" " " irr quad

$(x - r_1)(x - r_2)$

(3) 1 both same
(6) 1 both diff

$3 \cdot \frac{(6)}{2}$

(2) Use mod 2

Then $\bar{f}(x) = x^4 + x + 1$

so degree not reduced

then check $x = 0, 1$ - no root so

irr over $\mathbb{Z}_2$ $\xrightarrow{\text{Thm}}$ irr over $\mathbb{Q}$

so no

$(x^2+1)(x^2+1) = x^4+1 \mod 2$ (no)

$x^2 \cdot x^2 = x^4$ (no)

$x^2(x^2+1)$ $x^4 + x^2$ (no)

(x^2+x+1) x^2 (NO)
x^2+1 (NO)

x^2+x+1 $(x^2+x+1)(x^2+1) =$
$x^4 + x^3$
$(x^2+x+1)(x^2+x+1)$
$x^4 + x^2$ (NO)

$x^4+1 = (x^2+1)^2$ over $\mathbb{Z}_2$

$x^4+1 = (x^2+1)(x^2-1)$.

x^4-1

$(x^2+x-1)(x^2-x-1)$ $\mathbb{Z}_3$

x^4+1

$\mathbb{Q}[\sqrt{2}]$ $(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1) =$

x^4+1

x^4+1

$y = x^2$

y^2+1

$x^4+1 = (x-2)(x+2)g(x) \mod 17$

Show $x^4 + 15x^3 + 7$ irr over $\mathbb{Q}$

mod 2 $\quad x^4 + x^3 + 1$

Check possibilities for 2 quad factors

No linear factor, since 0, 1 not roots

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5) $x^3 + x^2 + x + 1$

$x^2(x+1) + (x+1) = (x^2+1)(x+1)$

$x^2 + 1 = (x+1)^2 \mod 2$

So $x^3 + x^2 + x + 1 = (x+1)^3 \mod 2$

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6) $(x-2)(x-3) = x^2 - 5x + 6$

mod 5 is $x^2 + 1$ ← use QF.

$$\frac{-0 \pm \sqrt{0^2 - 4}}{2} = \frac{\pm 2i}{2}$$

Find roots of $x^2+1 \bmod 5$

$x^2-5x+1+5 \equiv 0 \bmod 5$

$x^2-5x+6 \rightarrow \frac{5 \pm \sqrt{25-24}}{2}$

$a=1$
$b=-5$
$c=6$

$\frac{5 \pm 1}{2} \Rightarrow x = 3 \text{ or } 2$

$ax^2+bx+c \equiv 0 \pmod p$

$4a^2x^2+4abx+4ac \equiv 0 \pmod p$

$4a^2x^2+4abx \equiv -4ac$

$b^2 + 4a^2x^2 + 4abx \equiv b^2-4ac$

$(2ax+b)^2 \equiv b^2-4ac \bmod p$

$(2x-5)^2 \equiv 25-4\cdot1\cdot6 \equiv 1 \pmod p$

mod 7

$(x-4)(x-5) =$

$x^2 - 9x + 20$

$x^2 - 2x - 1 \equiv 0 \mod 7$

$a = 1$

$b = -2$

$c = -1$

$(2x-2)^2 \equiv 4+4 \equiv 1 \quad (7)$