

①

Given $|G| = 2^4 \cdot 3$ $\left\{ \begin{array}{l} |2\text{-SSG}| = 16 \\ |3\text{-SSG}| = 3 \end{array} \right.$

$N(2) \equiv 1 \pmod{2} \Rightarrow N(2) \text{ odd}$

also $N(2) | 48$ so $N(2) = \boxed{1, 3}$

$N(3) \equiv 1 \pmod{3} \Rightarrow N(3) = 3n+1$

for what n does $3n+1 | 48$

$n = 0, 1, 5 \Rightarrow N(3) = \boxed{1, 4, 16}$

Find compatible outcomes:

~~If $N(3) = 1$ then $16 \cdot 3 = 48$ el left
 so $N(2) = 15$ works $16 \cdot 2 = 32$ el left
 a solution is $N(3) = 1$ & $N(2) = 15$~~

$N(2) = 1$ then $15 \text{ el} + e_6$ forces

$N(3) =$ then $16 \cdot 2 \text{ el.} + e_6$

48 el tot.

(2)

$$N(2) = 3$$

$$N(3) = 1$$

~~48~~ 48 non-id + e_G
 2 non-id + e_G
48 el tot.

$$|A_5| = 2^2 \cdot 3 \cdot 5$$

$$|2\text{-SSG}| = 4$$

$$N(2) \equiv 1 \pmod{2} \quad N(2) \text{ odd}$$

3 non id
1 id

$$N(2) \mid 60 \quad 1, 3, 5, 15$$

$$N(3) \equiv 1 \pmod{3} \Rightarrow N(3) = 3n+1$$

(3-SSG) =
2 non id
1 id

$$3n+1 \mid 60 \Rightarrow N(3) = 1, 4, 7, 10$$

$$n = 0, 1, 3$$

$$N(5) \equiv 1 \pmod{5} \Rightarrow 5n+1$$

$$5n+1 \mid 60$$

$$n = 0, 1,$$

$$N(5) = 1, 6$$

3

$N(2)$ 1, 3, 5, 15

$N(3)$ 1, 4, 10

$N(5)$ 1, 6

Case 1 $N(5) = 1$ 4 non id 1 id

55 non id left

$N(3) = 1$ 2 non-id

53 non id

$N(5) = \text{DNE}$

Case 2 $N(5) = 1$

1 id
4 id
8

$N(3) = 4$

Case 3

~~$N(2) = 4$
 $N(3) = 10$~~

$N(5) = 1$

non id

4

$N(3) = 10$

20

2 3 5

2 3 5

any 1 1

any 4 1

any 10 1

any 4 6

(4)

$$N(5) = 6$$

24 ● non-id

$$N(3) = 4$$

8

$$\underline{N(2)} = 4$$

no!

$$N(5) = 6$$

24

$$N(3) = 10$$

20

$$N(2) = 5$$