

Suppose  $H \leq G$

Let  $K = xHx^{-1}$  for some  $x \in G$

Claim  $K \leq G$

Pick  $a, b \in xHx^{-1} \Rightarrow a = xh_1x^{-1}$   
 $b = xh_2x^{-1}$

Look @  $ab^{-1} = (xh_1x^{-1})(xh_2x^{-1})^{-1} =$   
 $(xh_1x^{-1})(xh_2^{-1}x^{-1}) =$   
 $\underbrace{xh_1x^{-1}xh_2^{-1}x^{-1}}_e = xh_3x^{-1}$   
 $= \underline{xh_3x^{-1}} \in xHx^{-1}$  ■

If  $H = xHx^{-1} \Rightarrow \underline{xH = Hx}$   
normal

Corollary: Self-conjugate iff normal ■

Sylow's 3<sup>rd</sup> Thm:  $|G| = p^k m$

$n_p \equiv N(p) = \#$  Sylow subgps of order  $p^k$

$N(p) \equiv 1 \pmod{p} \quad \# \quad N(p) \mid m$

(2)

$$|G| = |S_3| = 6 = 2 \cdot 3$$

2-SSG's ; 3-SSG's

$$H_1 \leq S_3 \quad H_1 = \{(1), (12)\}$$

$$H_2 = \{(1), (13)\}$$

$$H_3 = \{(1), (23)\}$$

$$\begin{aligned} & \overbrace{(13) \{(1), (12)\} (13)^{-1}}^{\leftarrow} = \\ & \{(1), (23)\} \text{ YES!} \end{aligned}$$

~~$$(13) \{(1), (12)\} (13)^{-1} = \{(1), (12)\}$$~~

~~$$(12) \{(1), (12)\} (12)^{-1} = \{(1), (12)\}$$~~

1	2	3
3	1	2
2	3	1

How many 2-SSG's

$$p^k m = |G|$$

$$(i) N(2) \equiv 1 \pmod 2$$

$$p=2 \quad m=3$$

$$(ii) N(2) | 3$$

$$N(2) = 1, 3$$

$$|G| = 40 = 2^3 \cdot 5$$

$$(i) N(2) \equiv 1 \pmod 2$$

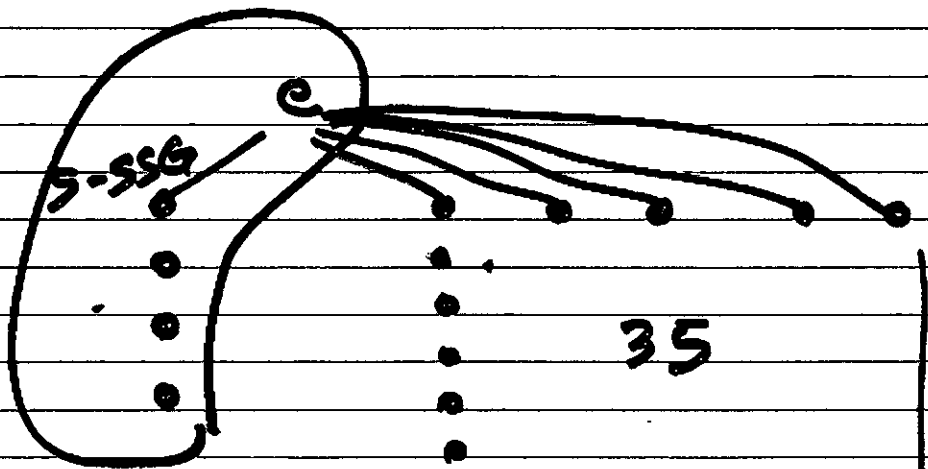
$$N(5) \equiv 1 \pmod 5$$

$$(ii) N(2) | 5$$

$$N(5) | 8 \quad \rightarrow \quad N(5) = 1$$

$$N(2) \equiv 1 \pmod 2$$

$$N(2) | 5$$



$\frac{1}{5}$  5-SSG  
 $\bullet$  2-SSGs