$a \in G$

$b = xax^{-1}$ for all $x \in G$

This exp appears $aaa^{-1} = a$

$\underline{xex^{-1} = e}$

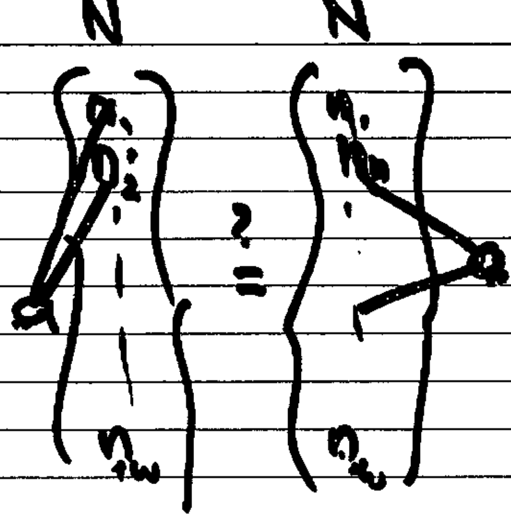
$xx^{-1}e = e$

$$
\begin{aligned}
x_1 a x_1^{-1} &= b_1 \\
x_2 a x_2^{-1} &= b_2 \\
a & \\
\vdots & \\
&= b_1 \\
&= b_1 \\
&= b_1 \\
a &= b_3 \\
\vdots & \quad \vdots \\
x_n a x_n^{-1} &= b_2
\end{aligned}
\in cl(a)
$$

xb_4x^{-1}

$\vdots$

$aN = Na$



$|G| = |C(a)| \cdot [G : C(a)]$

②

Thm 24.1 $|cl(a)| = [G : C(a)]$

$T(\underline{xC(a)}) \longmapsto \underline{xax^{-1}}$

Well-def'y:

Must show if $\boxed{x(C(a)) = y(C(a))}$ then

$xax^{-1} = yay^{-1}$

$y = xr \quad r \in C(a)$

$xH = H \Rightarrow x \in H$

$\underline{C(a) = x^{-1}yC(a)}$

$x^{-1}y \in C(a)$

$(x^{-1}y)a(x^{-1}y) = (x^{-1}y)a(y^{-1}x)$

$a = x^{-1}yay^{-1}x$

$xa = xx^{-1}yay^{-1}x = yay^{-1}x$

$\underline{xax^{-1}} = yay^{-1}(xx^{-1}) = \underline{yay^{-1}}$

Injectivity:

$$xax^{-1} = yay^{-1} \Rightarrow$$

$$y^{-1}xax^{-1} = ay^{-1}$$

$$\underline{y^{-1}xax^{-1}}\,y = ay^{-1}y = \underline{a}$$

$$\downarrow$$

$$(y^{-1}x)a(y^{-1}x)^{-1} = a$$

$$y^{-1}x \in C(a)$$

$$\boxed{\begin{matrix} a \in H \\ aH = H \end{matrix}}$$

$$y^{-1}xC(a) = C(a)$$

$$yy^{-1}xC(a) = yC(a)$$

$$\therefore\ xC(a) = yC(a)\ \checkmark$$

So T is injection

Surjectivity: by construction

So T is bijective $\Longrightarrow$

$$|cl(a)| = [G:C(a)]$$

■

Class Eqn.

Thm: $a \in G \quad |cl(a)| \mid |G|$

$|cl(a)|$

$$|G| = \sum_{a \in A} [G : C(a)]$$

A is set of distinct conj. class reps.