Proposition 1: There exists a unique set with no elements.

Proposition 2: (Existence of intersection) If P and Q are sets, then there is a set R such that $x \in R$ if and only if $x \in P$ and $x \in Q$.

Proposition 3: Given A, there is a unique B such that $x \in B$ if and only if $x \in A$ and $P(x)$ is true.

Proposition 4: For any property P, $\{x \in \varnothing | P(x) \text{ is true}\} = \varnothing$.

Proposition 5: The set given by the Axiom of Pair is unique.

Proposition 6: The set given by the Axiom of Union is unique.

Proposition 7: The set given by the Axiom of Power Set is unique.

Proposition 8: The set of all x such that $x \in A$ and $x \notin B$ exists.

Proposition 9: The Weak Axiom of Existence (some set exists) implies the Axiom of Existence.

Proposition 10: $\wp(X) \subseteq X$ is never true.

Proposition 11: Given the set A, the set $\{x | x \notin A\}$ does not exist.

Proposition 12: Given the nonempty system of sets S, $\bigcap S$ exists.

Proposition 13: $\bigcap \varnothing$ is the universe.

Proposition 14: Given a set S, $\wp(S)$ is a commutative ring with identity under the operations $\triangle$ and $\cap$

Proposition 15: $\bigcup S^c = (\bigcap S)^c$

Proposition 16: $\bigcap S^c = (\bigcup S)^c$

Proposition 17: Let R be a binary relation, both $domR$ and $ranR$ exist.

Proposition 18: If $a \in A$ and $b \in A$, then $(a,b) \in 2^{2^A}$.

Proposition 19: $(a,b) = (b,a)$ implies $a = b$.

Proposition 20: If S and R are relations on A, then $S \circ R$ exists.

Proposition 21: The following are true for relation R and sets $A, B \subseteq domR$: (i) $R(A \cup B) = R(A) \cup R(B)$, (ii) $R(A \cap B) \subseteq R(A) \cap R(B)$, (iii) $R(A - B) \supseteq R(A) - R(B)$.

Proposition 22: Composition of relations on a given set is associative.

Proposition 23: $\exists X$ such that $X^3 \neq X \times X^2$

Proposition 24: $A \times B = \varnothing$ if and only if at least one of A and B is empty.

Proposition 25: If f is a function, the following are true: (i) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, (ii) $f^{-1}(A \cap B) \subseteq f^{-1}(A) \cap f^{-1}(B)$, (iii) $f^{-1}(A - B) \supseteq f^{-1}(A) - f^{-1}(B)$.

Proposition 26: Every system of sets S can be indexed by a function $f : I \to S$, where I is a suitable index set

Proposition 27: $\bigcup_{\alpha \in \bigcup S} = \bigcup_{C \in S}(\bigcup_{\alpha \in C} F_\alpha)$

Proposition 28: $\bigcap_{\alpha \in \bigcup S} = \bigcap_{C \in S}(\bigcap_{\alpha \in C} F_\alpha)$

Proposition 29: If f is a function, $f(\bigcup_{\alpha \in A} F_\alpha) = \bigcup_{\alpha \in A} f(F_\alpha)$

Proposition 30: If f is a function, $f(\bigcap_{\alpha \in A} F_\alpha) \subseteq \bigcap_{\alpha \in A} f(F_\alpha)$

Proposition 31: The composition of injective functions is injective and the composition of surjective functions is surjective.

Proposition 32: If $F : X \to X$ is injective (resp. surjective) then it is surjective (resp. injective) provided ______________.

Proposition 33:There is no set that contains all sets equipotent to the set $A \neq \varnothing$. Hint:

Russell paradox.

Proposition 34: Let $S(x)$ be the successor of x. There does not exist z such that $x \subset z \subset S(x)$.

Proposition 35: Let $P(x)$ be a property (possibly with parameters). If $P(0)$ is true and for all n, $P(n)$ implies $P(n+1)$, then $P(n)$ is true for all $n \in \mathbb{N}$. Can't assume $\mathbb{N}$ is well-ordered..yet.

Proposition 36: Let $P(x)$ be a property (possibly with parameters). Assume that for all $n \in \mathbb{N}$, if $P(k)$ holds for $k < n$, then $P(n)$ is true. Then $P(n)$ is true for all $n \in \mathbb{N}$.

Proposition 37: Let $P(x,y)$ be a property. If $P(k,l)$ holds for all $k,l \in \mathbb{N}$ such that $k < m$ or ($k = m$ and $l < n$) implies $P(m,n)$ is true, then $P(m,n)$ is true for all $m,n \in \mathbb{N}$.

Proposition 38: Let $P(x)$ be a property (possibly with parameters). If $P(k)$ is true and for all $n > k$, $P(n)$ implies $P(n+1)$, then $P(n)$ is true for all $n > k$.

Proposition 39: Assuming $\mathbb{N}$ is well ordered by $<$, let $P(x)$ be a property (possibly with parameters). If $P(0)$ is true and for all n, $P(n)$ implies $P(n+1)$, then $P(n)$ is true for all $n \in \mathbb{N}$.

Proposition 40: Every well-ordered set is linearly ordered.

Proposition 41: $\mathbb{N}$ is well ordered by $<$.

Proposition 42: There is no function $f : \mathbb{N} \to \mathbb{N}$ such that $f(n) > f(n+1)$.

Proposition 43: $\mathbb{N}$ is equipotent to infinitely many of its proper subsets.

Proposition 44: Given $n \in \mathbb{N}$, there is no $k \in \mathbb{N}$ such that $n < k < n+1$.

Proposition 45: For all $m,n \in \mathbb{N}$, if $m < n$, then $m+1 < n$. Hint: use Proposition 44.

Proposition 46: The successor mapping $S(n)$ is injective.

You may assume ..Theorem: Let $a : P \to A$ and $g : P \times A \times \mathbb{N}$ be functions. There exists a unique function $f : P \times \mathbb{N} \to A$ such that (i) $f(p,0) = a(p)$ for all $p \in P$ and (ii) $f(p,n+1) = g(p,f(p),n)$ for all $n \in \mathbb{N}$ and $p \in P$.

Proposition 47: There is a unique binary operation $+ : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ such that (i) $+(m,0) = m$ for all $m \in \mathbb{N}$ and (ii) $+(m,n+1) = +(m,n)+1$ for all $m,n \in \mathbb{N}$

Proposition 48: The operation of addition defined in Prop. 47 is commutative.

Proposition 49: (Refer to the Peano Axioms in the ZFC summary) $\mathbb{N}$ satisfies the Peano axioms.

Proposition 50: If $m,n \in \mathbb{N}$, then $m \leq n$ if and only if there exists a unique $k \in \mathbb{N}$ such that $n = m+k$. Define $-(n,m) = k$.

Proposition 51: There exists a unique binary operation $\cdot : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ such that $m \cdot 0 = 0$ for all $m \in \mathbb{N}$ and $m \cdot (n+1) = m \cdot n + m$ for all $m,n \in \mathbb{N}$. Recall: addition is now defined.

Proposition 52: Multiplication (the operation in Prop. 51) is commutative.

Proposition 53: Define $E : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ as follows: (i) $E(m,0) = 1$, for all $m \in \mathbb{N}$ (even 0!..don't tell anyone), (ii) $E(m,n+1) = m \cdot E(m,n)$. Prove the usual laws of exponents.

Proposition 54: Given a set of n-tuples $R \subseteq X^n$ for some nonempty set X, show that R induces a relation with arity n.

Proposition 55: The structure $(\mathbb{N},<)$ has only one automorphism.

Proposition 56: The structure $(\mathbb{Z},<)$ has infinitely many automorphisms.

Transfinite Arithmetic:

Proposition 57: Cardinal addition is commutative.

Proposition 58: Cardinal addition is associative.

Proposition 59: Cardinal multiplication is commutative.
Proposition 60: Cardinal multiplication is associative.
Proposition 61: Cardinal multiplication distributes over cardinal addition.
Proposition 62: $\aleph_0 + \aleph_0 = \aleph_0$ and $\aleph_0 \cdot \aleph_0 = \aleph_0$
Proposition 63: $\aleph_0 + n = \aleph_0$ and $\aleph_0 \cdot n = \aleph_0$ for any $n \in \mathbb{N}$
Proposition 64: $\aleph_0 + \aleph_1 = \aleph_1$ and $\aleph_0 \cdot \aleph_1 = \aleph_1$
Proposition 65: For cardinals κ, λ, μ it is true that $\kappa^{\lambda+\mu} = \kappa^\lambda \cdot \kappa^\mu$
Proposition 66: For cardinals κ, λ, μ it is true that $(\kappa^\lambda)^\mu = \kappa^{\lambda\mu}$
Proposition 67: $\aleph_0^{\aleph_0} > \aleph_0$
Proposition 68: Show that every $n \in \mathbb{N}$ is an initial ordinal
Proposition 69: Every well-orderable set is equipollent to an ordinal number
Proposition 70: Every well-orderable set is equipollent to a unique initial ordinal number
Proposition 71: The Hartogs number for any set is an initial ordinal
Proposition 72: For any ordinal β, $\beta \cdot 1 = \beta$
Proposition 73: For any ordinal β, $\beta \cdot 2 = \beta + \beta$
Proposition 74: $2 \cdot \omega \neq \omega \cdot 2$
Proposition 75: $2 \cdot \omega = \omega$
Proposition 76: For any ordinal β, $\beta^2 = \beta \cdot \beta$
Proposition 77: $1^\omega = 1$
Proposition 78: $2^\omega = 3^\omega$
Proposition 79: $\omega^\omega > \omega$
Proposition 80: Find the least ordinal ξ such that $\omega + \xi = \xi$
Proposition 81: Every strong limit cardinal is a limit cardinal
Proposition 82: Every limit cardinal is a strong limit cardinal if GCH holds
Proposition 83: Every successor cardinal $\aleph_{\alpha+1}$ is regular
Proposition 84: There exist arbitrarily large singular cardinals
Proposition 85: There are arbitrarily large singular cardinals $\aleph_\alpha$ such that $\aleph_\alpha = \alpha$
Proposition 86: The cofinality of α is a limit ordinal
Proposition 87: An ordinal is never less than its cofinality.
Proposition 88: (Test for singularity) A cardinal $\aleph_\alpha$is singular if $cf(\omega_\alpha) < \omega_\alpha$ and regular if $cf(\omega_\alpha) = \omega_\alpha$