

CALCULUS 3 - FALL 2017 - HOMEWORK 4 (Quadrics) - Solutions

1) What is the equation of an ellipsoid centered @ (1,2,3) with semi-axes 2,2, and 5 in the x, y, and z directions, respectively?

From the chart on p.830 in Stewart, an ellipsoid has equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, where $a, b,$ and c determine the extents (semi-axes) in the respective directions. So the basic ellipsoid with the given extents would be $\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{25} = 1$. Now we move it to (1,2,3) the usual way: $\frac{(x-1)^2}{4} + \frac{(y-2)^2}{4} + \frac{(z-3)^2}{25} = 1$

2) Find an equation for the surface obtained by rotating the parabola $y = x^2$ around the y-axis.

As y increases, the sections perpendicular to the y-axis are circles with radius \sqrt{y} , so the equation is $y = x^2 + z^2$, and the figure is a paraboloid.

3) Find an equation for the surface obtained by rotating the line $x = 3y$ about the x-axis.

The line given by $y = \frac{1}{3}x$ spun around the x-axis creates a circle with radius $\frac{x}{3}$ for fixed x , so $y^2 + z^2 = \left(\frac{x}{3}\right)^2$... this is a cone with axis along the x-axis.

4) Find an equation for the surface consisting of all points P for which the distance from P to the x-axis is twice the distance from P to the yz-plane.

Consider a circle parallel to the yz-plane at some fixed value of x . All the points of the circle are equidistant from the yz-plane. If the radius of the circle for some fixed x is $2x$, then the condition defining the surface is satisfied. So $y^2 + z^2 = 4x^2$.

5) Identify the surface in (4)

This is a cone opening along the x-axis.

6) Identify the surface given by $x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$

$x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = x^2 - 2x + 1 - (y^2 - 2y + 1) + z^2 + 4z + 4 = 2$, so
 $(x - 1)^2 - (y - 1)^2 + (z + 2)^2 = 2$. *In standard form this is*
 $\left(\frac{x-1}{\sqrt{2}}\right)^2 - \left(\frac{y-1}{\sqrt{2}}\right)^2 + \left(\frac{z+2}{\sqrt{2}}\right)^2 = 1$. *This a hyperboloid of one sheet opening along the y-axis.*