

CALCULUS 3 - FALL 2017 - HOMEWORK 3 (Spatial Geometry)

1) What is the equation of the line that goes thru  $(1, 3, 4)$  and  $(-1, 4, 2)$  in vector form?

*Subtract respective coordinates to get direction vector, then start at either of the points.*

$$r(t) = \langle 1, 3, 4 \rangle + \langle 2, -1, 2 \rangle t$$

2) What are the equations of the line in (1) in parametric and symmetric forms?

$$\text{Equations for coordinates are: } x(t) = 1 + 2t \quad y(t) = 3 - t \quad z(t) = 4 + 2t$$

$$\text{Solving each for } t \text{ and then equating: } \frac{x-1}{2} = 3-y = \frac{z-4}{2}$$

3) What is equation of the plane thru  $(2, 4, -5)$  perpendicular to the line connecting the origin to the point  $(6, 4, -5)$ ?

*Direction vector for plane is the vector  $\langle 6, 4, -5 \rangle$ , and generic vector in plane is  $\langle x-2, y-4, z+5 \rangle$ . Dotting and setting equal to zero gives  $6(x-2) + 4(y-4) - 5(z+5) = 0$ . You can put in standard form if you like.*

4) What is the dihedral angle between the planes given by  $x + 2y - 3z = 1$  and  $2x - y - 5z = 7$ ?

*Angle between planes is same as angle between normals. Normals are  $\langle 1, 2, -3 \rangle$  and  $\langle 2, -1, -5 \rangle$ . Then  $\theta = \arccos\left(\frac{\langle 1, 2, -3 \rangle \cdot \langle 2, -1, -5 \rangle}{\|\langle 1, 2, -3 \rangle\| \|\langle 2, -1, -5 \rangle\|}\right) = \arccos\left(\frac{15}{20.5}\right) = 43 \text{ degrees}$*

5) What is the distance between the plane given by  $3x - 3y + 5z = 11$  and the point  $(7, 4, -3)$ ?

*Point to plane distance is the projection onto the plane's direction vector of any vector connecting the plane and point. The point  $(1, -1, 1)$  is in the plane (plug in the coordinates and check), so the vector  $\langle 6, 5, -4 \rangle$  goes from that point in the plane to the given point outside the plane. Then reading the coefficients of the plane equation in standard form into a direction vector, we have  $\langle 3, -3, 5 \rangle$ . Dotting  $\langle 6, 5, -4 \rangle$  and  $\langle 3, -3, 5 \rangle$  we get  $-17$ , and then we need to divide this by the length of the direction vector to get the scalar projection of the point-to-point vector. So the distance is  $\frac{17}{\sqrt{43}} = 2.59 \text{ units}$ .*

6) What is the distance between the x-axis and the line given by  $r(t) = \langle 1, 2, 3 \rangle + \langle 1, 1, 4 \rangle t$ ?

The  $x$ -axis has equation (as a line)  $r(s) = \langle 1, 0, 0 \rangle s$ , where we use a different parameter than that for the second line. We imagine these two lines being in parallel planes with the direction vectors for the planes being mutually perpendicular to the lines. So we need a vector for that...the cross product will do.  $n = \langle 1, 0, 0 \rangle \times \langle 1, 1, 4 \rangle = \langle 0, -4, 1 \rangle$ . The distance between the lines is the same as the distance between the planes, so we proceed as in (5). Letting  $t$  and  $s$  be zero gives particularly simple points in both planes, namely the origin and  $(1, 2, 3)$ . Now project  $\langle 1, 2, 3 \rangle$  onto the normal  $\langle 0, -4, 1 \rangle$  to get  $-5$ , then the distance is  $\frac{5}{\sqrt{17}}$ .