CALCULUS 3 - FALL 2017 - HOMEWORK 2 - Solutions

1) Line $\ell_1$ connects the origin to the point $(2,5,7)$ and line $\ell_2$ connects the point $(2,5,7)$ to the point $(-1,3,3)$. What is the angle between the lines at their intersection?

*Let* $A = \langle 2,5,7\rangle$ *and* $B = \langle -3,-2,-4\rangle$.

$$\theta = \arccos\frac{A\cdot B}{|A||B|} = \arccos\frac{-6-10-28}{\left(\sqrt{78}\right)\left(\sqrt{29}\right)} = \arccos\frac{-44}{\sqrt{2262}} = 157.7 \text{ degrees}$$

2) A laser beam parallel to the xy plane strikes the xz plane and is reflected onto the yz plane. Show that the beam returns parallel to its original path.

*See class notes for figure showing angles of incidence and reflection.*

3) What is the projection of the vector $\langle 4,6,-2\rangle$ onto the vector $\langle 2,2,5\rangle$?

$$\frac{\langle 4,6,-2\rangle\cdot\langle 2,2,5\rangle}{|\langle 2,2,5\rangle|} = \frac{8+12-10}{\sqrt{33}} = 1.74$$

4) What is volume of the parallelepiped (fancy name for deformed shoe box) determined by the vectors $\langle 1,2,3\rangle$, $\langle -1,1,2\rangle$, and $\langle 2,1,4\rangle$?

$$\langle 1,2,3\rangle\cdot\langle -1,1,2\rangle\times\langle 2,1,4\rangle = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 2 & 1 & 4 \end{vmatrix} = (1)(4-2)-(2)(-4-4)+(3)(-1-2), \text{ which}$$

*yields* $2+16-9 = 9$

5) What is the area of a triangle formed by the two vectors $\langle 2,4,6\rangle$ and $\langle -1,-1,7\rangle$?

$$\text{Area} = \frac{1}{2}|\langle 2,4,6\rangle\times\langle -1,-1,7\rangle| = \frac{1}{2}\begin{vmatrix} i & j & k \\ 2 & 4 & 6 \\ -1 & -1 & 7 \end{vmatrix} = \frac{1}{2}|34i-20j+2k| = 19.8$$

6) Using the trick of embedding a tetrahedron in a cube, what is the formula for the volume of a tetrahedron with side 1?

*See class notes for drawing.*