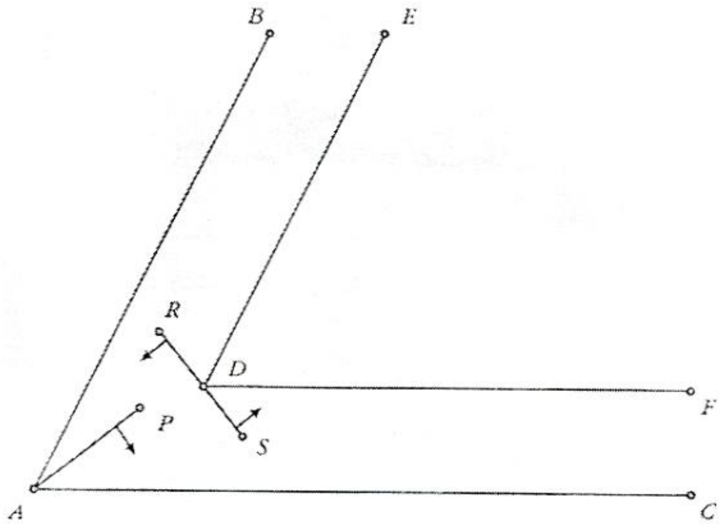
College Mathematics Journal

Problem 661 - November 1999

Solution:

Using the notation given in Figure 1 of the problem, it is clear that no ladder of length greater than AD could be gotten around the corner by rotating about the outer corner A. Likewise, any ladder rotated about the inner corner must at some point be perpendicular to $\overline{AD}$. For this ladder, extend $\overline{RDS}$ to intersect both $\overline{AB}$ and $\overline{AC}$, calling the points of intersection R' and S', respectively. By symmetry, $\overline{R'D} = \overline{DS'}$. Then $R'S'$ must obviously be the maximum length for a ladder rotated about D. If $AD = R'S'$, then evidently $\alpha = 2\arctan(\frac{1}{2}) = .927$ (about 53^0). There are no other values of $\alpha \in (0, \pi)$ for which the limiting lengths from the two methods are equal, since AD is fixed for a given hallway width, and $R'S'$ varies monotonically from 0 to ∞ as α goes from 0 to π. Moreover, by similarity, the solution is independent of hallway width.



Solution by Robert Patenaude, College of the Canyons, Valencia, CA

Place A at the origin of a coordinate system and place $D = (0, b)$ on the positive y-axis. Because the diagram is symmetric about the y-axis, AC has equation $y = mx$, where $m = \cot\left(\frac{\alpha}{2}\right) > 0$, and AB has equation $y = -mx$. A line $y = nx + b$ through D with slope, n, $-m < n < m$, intersects AC at $H = \left(\frac{b}{m-n}, \frac{mb}{m-n}\right)$, and intersects AB at $G = \left(\frac{-b}{m+n}, \frac{mb}{m+n}\right)$. The length c_0 of the longest ladder that can be swung around D is the minimum of $c = |GH| = \frac{2mb\sqrt{n^2+1}}{m^2 - n^2}$ over n in $(-m, m)$. We conclude from

$$\frac{dc}{dn} = \frac{2mnb(m^2 + n^2 + 2)}{(m^2 - n^2)^2\sqrt{n^2+1}}$$

that c has a minimum of $c_0 = \frac{2b}{m}$ uniquely assumed when $n = 0$. But $b = |AD|$ is the length of the longest ladder that can be swung about A, so the given condition $b = c_0$ becomes $b = \frac{2b}{m}$, whence $m = 2$, or $\alpha = 2\arctan(\frac{1}{2})$.

Also solved by HERB BAILEY, Rose-Hulman Institute of Technology; THE BOOKERY PROBLEM GROUP, Walla Walla, WA; ROBERT D. CRISE, JR., Highland, CA; DANIELE DONINI, Bertinoro, Italy; JAMES DUEMMEL, Bellingham, WA; NEIL EKLUND, Centre C.; FLORIDA GULF COAST U. PROBLEM GROUP, Florida Gulf Coast U.; ALAN GORFIN and DICK PELOSI (jointly), Western New England C.; RICKY IKEDA, Leeward C. C.; HOYT JOLLY, JOHN MISINCO, TAM TRAN, students, Georgia Institute of Technology; PHILIP OPPENHEIMER, South Norwalk, CT; FARY SAMI, Harford C. C.; and the proposer. Two solvers interpreted the problem in a more restricted way, arriving at a different solution.

Probabilities and Expected Values in a Coin Tossing Game

662. *Proposed by Bradley A. Warner and Bradford Kline, United States Air Force Academy, Colorado Springs, CO*

Two players use a coin that lands heads with probability p to play a game that consists of a sequence of rounds. In each round, the first player tosses the coin until a head appears. Then the second player tosses the coin until a head appears. If the