Test Review

1. Let (a,b) be a basic set in $\mathcal{U}$ on $\mathbb{R}$.

Note $\bigcup_{i=1}^{\infty} [a_i - \frac{1}{n}, b) = (a,b)$

because $a_i \notin [a_i - \frac{1}{n}, b)$ $\forall i \in \mathbb{N}$.

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2. $$\phi(x) = \frac{x}{1+x^2} \text{ or } \frac{x}{1+|x|}$$

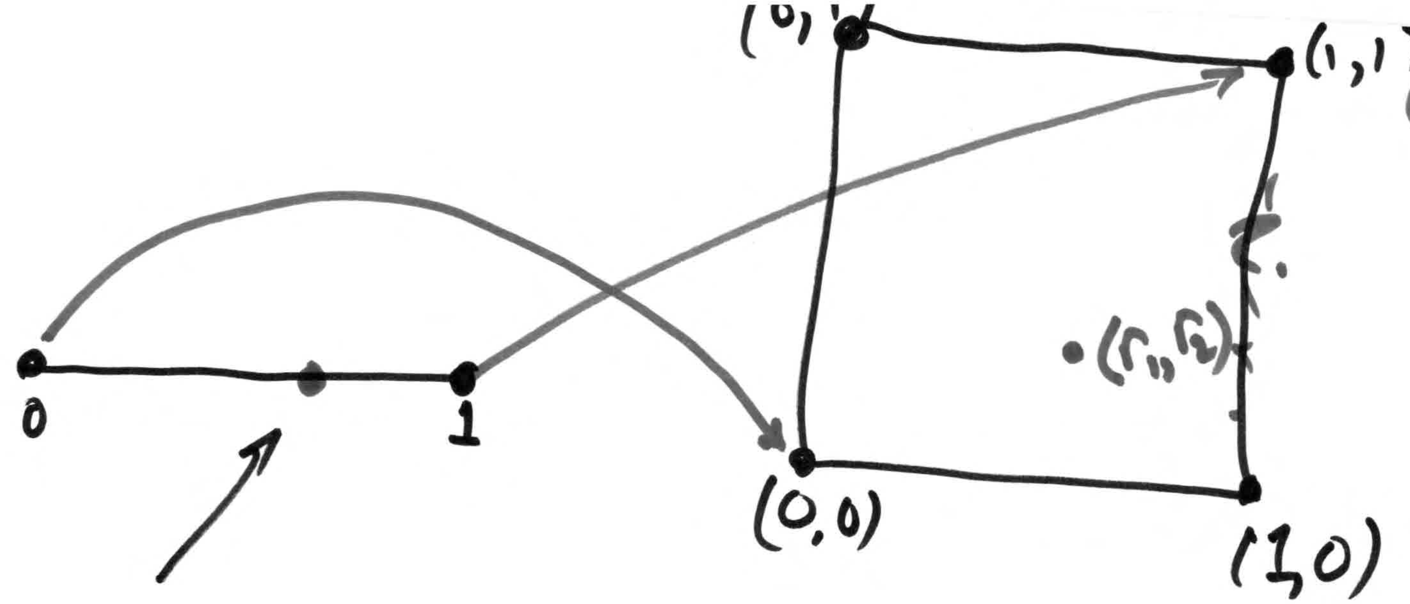
$$\phi(-2) = \frac{-2}{1+(-2)^2} = -\frac{2}{5}$$

$$\frac{x}{1-x^2} \qquad \Phi(x) = $$

$\frac{2}{\pi} \arctan x$

$$\psi(y) = \frac{y}{1-y^2}$$

$y \in (-1,1) \to$



$$r = 0.d_1 d_2 d_3 d_4 \cdots d_{2k} d_{2k+1} \cdots \to$$

$$r_x = 0.d_1 d_3 d_5 \cdots d_{2k+1} \cdots \to$$

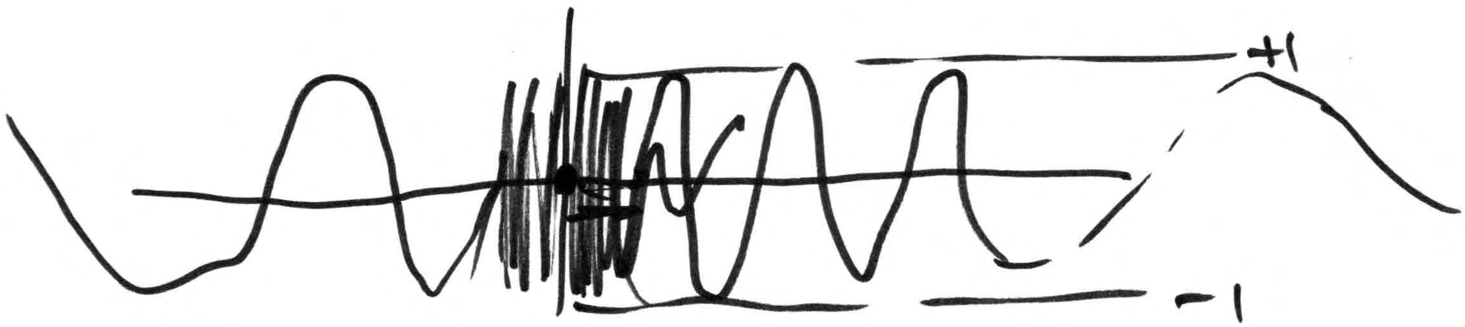
$$r_y = 0.d_2 d_4 d_6 \cdots d_{2k} \cdots \to$$

$$0 = 0.000 \to$$

$$1 = .999 \cdots \to$$

$$r_x \quad \underline{0.99999} \cdots \to$$

$$f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$



$$g(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

$g \in C(\mathbb{R})$

Claim $\lim_{x \to 0} g(x) = 0$

Show:

Fix $\varepsilon > 0$. Need $\delta(\varepsilon) > 0 \ni |x - 0| < \delta(\varepsilon)$

$\Rightarrow \left| x \sin\left(\frac{1}{x}\right) - 0 \right| < \varepsilon$

Want $\delta(\varepsilon) \ni |x| < \delta(\varepsilon)$

$$|x \sin(\tfrac{1}{x})| < \varepsilon$$

Try $\delta(\varepsilon) = \varepsilon$.

If $|x| < \delta(\varepsilon)$ then since $|x \sin(\tfrac{1}{x})| =$

$$|x| \underbrace{|\sin(\tfrac{1}{x})|}_{\text{never greater than } 1} \leq |x| \leq \delta(\varepsilon) \cancel{= \varepsilon}$$

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$$g(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & \text{else} \end{cases}$$

if $x \neq 0$ $g'(x) = x^{\cancel{3}}\overset{1}{} \cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{\cancel{x^2}}\right) +$

$g'(0) = 0$

$\lim_{x \to 0} g'(x)$ exists

$3x^2 \cdot \sin\left(\frac{1}{x}\right)$

$$g'(x) = 3x^2 \sin\left(\frac{1}{x}\right) - x\cos\left(\frac{1}{x}\right) \text{ * cont.}$$

$$\lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} = \frac{\cancel{x^3} \sin\left(\frac{1}{x}\right) - 0}{\cancel{x} - 0}$$

$$= x^2 \sin\left(\frac{1}{x}\right) \longrightarrow 0 \text{ as } x \longrightarrow 0$$

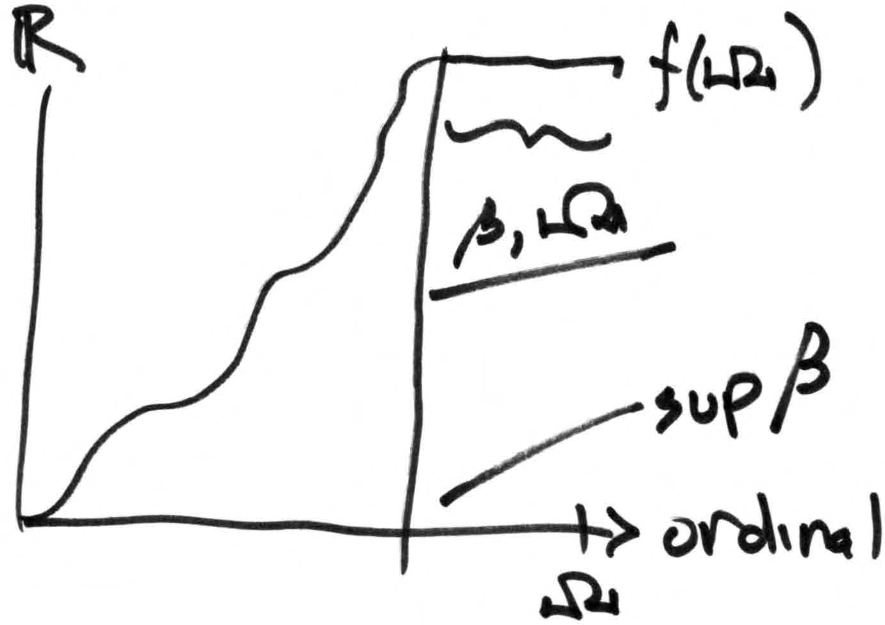
$$= \underbrace{x}_{\to 0} \underbrace{\left(x \sin\left(\frac{1}{x}\right)\right)}_{\to 0}$$

$|\sin\left(\frac{1}{x}\right)| \leq 1$

$\left|\sin\left(\frac{1}{x}\right)\right| \leq 1$ or $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$

$f : \cancel{\mathbb{R} \to [1, \Omega_1]}$ $[1, \Omega_1] \to \mathbb{R}$

f is eventually constant



Look @ $\underline{f(\Omega_1)} \in \alpha$ (β, Ω_1)

Let $\alpha \in U$ $U \in \mathcal{U}$

$f^{-1}(U)$?