SPRING - 2017 - CALCULUS 3 - TEST #1A

Vectors are in boldface. In particular, $\mathbf{r}(t)$ is a vector describing a smooth non-self-intersecting curve. $\|\mathbf{A}\|$ is the length of vector $\mathbf{A}$
True or false:

Make sure your name is on the scantron sheet and you mark where you started (answer #1).

1) Every directed line segment in $\mathbb{R}^3$ corresponds to a vector
2) A vector is uniquely determined by its components relative to a basis
3) Vectors may always be written as integer multiples of basis vectors – could be any real
4) Vectors may be added or subtracted
5) The dot product of two vectors is a vector – scalar
6) The norm of a vector is its length squared – not squared
7) The vector triple product is $\mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$ – scalar triple product
8) There is a vector perpendicular to itself – 0
9) $\mathbf{A} \times \mathbf{B}$ is perpendicular to $\mathbf{A}$
10) $\|\mathbf{A} \times \mathbf{B}\| = \|\mathbf{A}\| \, \|\mathbf{B}\| \cos\theta$ – $\sin\theta$
11) Vectors obey the triangle inequality
12) A triangle with sides $\mathbf{A}$ and $\mathbf{B}$ has area $\frac{1}{2}(\mathbf{A} \times \mathbf{B})$ – $\|A \times B\|$
13) Parallel vectors have zero dot product – perpendicular
14) $\mathbf{A} \cdot \mathbf{A}$ is the area of a square with side $|\mathbf{A}|$
15) A plane is determined by a vector perpendicular to the plane – need point on plane
16) The equation of a plane thru the point $\mathbf{r}_0$ parallel to $\mathbf{v}$ is $(\mathbf{r} - \mathbf{r}_0) \cdot \mathbf{v} = 0$ – perpendicular
17) The volume of a parallelepiped with adjacent sides $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ is $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C}$ – $|\cdot|$
18) The dot product distributes over vector addition
19) $\mathbf{A} \times (\mathbf{B} \times \mathbf{A}) = -(\mathbf{A} \times \mathbf{A}) \times \mathbf{B}$ – $A \times A = 0$
20) Lines can be nonparallel and nonintersecting in $\mathbb{R}^2$
21) $\mathbf{r}'(t)$ is a vector tangent to the path vector $\mathbf{r}(t)$
22) The unit tangent vector $\mathbf{T}(t) = \mathbf{r}'(t)/\|\mathbf{r}(t)\|$ – $\|r'(t)\|$
23) The unit binormal vector $\mathbf{B}(t) = \mathbf{N}(t) \times \mathbf{T}(t)$, where $\mathbf{N}(t)$ is the unit normal vector – $T \times N$
24) The unit normal vector $\mathbf{N}(t) = \mathbf{r}''(t)/\|\mathbf{r}''(t)\|$ – no, but close
25) The product of curvature and radius of curvature is constant at any point of $\mathbf{r}(t)$
26) $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 2$ is the equation of an ellipsoid
27) The graph of $\frac{x^2}{4} - \frac{y^2}{9} - \frac{z^2}{25} = 1$ consists of two disjoint sheets
28) $-z + x^2 + y^2 = 0$ describes a circular paraboloid
29) The graph of $\frac{x^2}{4} + \frac{y^2}{9} = \frac{z^2}{25} + 1$ is a cone – no "1"
30) Differential arc length $ds = \|\mathbf{r}(t)\|dt$ – need $r'(t)$
31) Arc length is given by $\int_a^b \|\mathbf{r}(t)\|ds$ – dt
32) If $\kappa(t) = 2$ for all t, then $\mathbf{r}(t)$ describes a circle of radius 2 – radius $1/2$
33) The derivative of a cross product is the cross product of derivatives – need product rule

34) The cross product is commutative – anti-

35) $\mathbf{r}(t) = \langle \cos t, \sin t, 1 \rangle$ describes a helix of constant pitch – need t

36) $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ describes the "twisted cubic"

37) The general equation for a plane can be written $ax^2 + by^2 + cz^2 = d$ – linear, not quadratic

38) Three planes can intersect in a point

39) Three points determine a unique plane – non-collinear

40) An acceleration vector is always parallel to its corresponding position vector – sometimes

41) For a vector of constant length, $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 0$ $\|r(t)\|' = 0$

42) $\mathbf{r}'(t)$ is the speed vector – no vector

43) The osculating plane contains $\mathbf{B}(t)$ – T and N

44) The normal plane is perpendicular to $\mathbf{T}(t)$ – N and B

45) Orthogonal means the same as normal

46) $\langle a,b,c \rangle \cdot \langle d,e,f \rangle = \langle ad, be, cf \rangle$ – no vector

47) $\langle 2\cos 3t, 3\sin 3t, 3 \rangle = \mathbf{r}(t)$ describes an ellipse in $\mathbb{R}^3$

48) $\frac{d}{dt}\langle e^{2t}, e^{3t}, e^4 \rangle = \langle 2e^{2t}, 3e^{3t}, 4e^4 \rangle$ – $(e^4)' = 0$

49) The unit normal vector always points at the center of curvature

50) Direction cosines are the angles a vector makes with the three coordinate axes – cosines of those angles