

## SPRING 2017 - ALGEBRA 2 - TEST #1A

No references. T/F two points each, scaled to 50 total

True or false:

- F 1) A commutative ring with no zero divisors is an integral domain - *need unity*
- 2) An ideal is an absorbing subring
- 3)  $\mathbb{Q}[x]$  is an integral domain
- F 4) Every subring of an integral domain is an ideal -  *$\langle 2 \rangle \subset \mathbb{Z}[x]$*
- 5) Every proper ideal of a ring has no units
- 6) An integral domain must have at least two elements
- 7) Cancellation holds in a Bézout domain
- 8) If  $u$  is a unit, so is  $-u$
- 9) A noncommutative ring can have a commutative subring
- F 10)  $\mathbb{Z}_4$  is an integral domain  *$2 \cdot 2 = 0$*
- 11) If  $D$  is an integral domain, so is  $D[x]$
- 12) If  $D$  is an integral domain,  $D[x][y] = D[x,y]$
- 13) If  $I$  is an ideal of a commutative unital ring  $R$ , then  $R/I$  is a factor ring
- F 14) If  $I$  is a prime ideal of a ring  $R$ , then  $R/I$  is an integral domain  *$R$  needs comm, unity*
- F 15) If  $I$  is a maximal ideal of a commutative ring  $R$ , then  $R/I$  is a field *ditto*
- F 16) If  $I$  is a prime ideal of  $R$ , then  $R/I$  is an integral domain *same as 14*
- F 17) If  $I$  is a prime ideal of a commutative unital ring  $R$ , then  $R/I$  is a field - *need maximal*
- 18) A field always has a maximal ideal
- F 19) If  $I$  is a maximal ideal of a commutative unital ring, then  $1 \notin R$  *nonsense*
- (+1) 20) Every prime ideal of a commutative unital ring is maximal
- F 21) An integral domain can have characteristic 6 - *0 or prime*
- 22)  $8\mathbb{Z}$  has no zero divisors
- F 23)  $\mathbb{Z}_8$  has no zero divisors -  *$4 \cdot 2 = 0$*
- F 24) A prime ideal can be the whole ring - *proper*
- F 25) The smallest ring is  $\mathbb{Z}_2$   *$\{0\}$*
- 26) Fields always have at least two ideals
- 27) If  $ab \in M$ , an ideal, implies  $a \in M$  or  $b \in M$ , then  $M$  is prime
- 28) If  $A, B$ , and  $P$  are ideals and  $AB \subset P$  implies  $A \subset P$  or  $B \subset P$ , then  $P$  is prime
- 29)  $\{0\}$  is always an ideal for any ring
- 30) A domain may have no non-trivial proper ideals